Optics II:
practical photographic lenses

CS 178, Spring 2014

Begun 4/10/14, finished 4/15

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Outline

- why study lenses?
- thin lenses
  - graphical constructions, algebraic formulae
- thick lenses
  - center of perspective, 3D perspective transformations
- depth of field

- aberrations & distortion
- vignetting, glare, and other lens artifacts
- diffraction and lens quality
- special lenses
  - telephoto, zoom

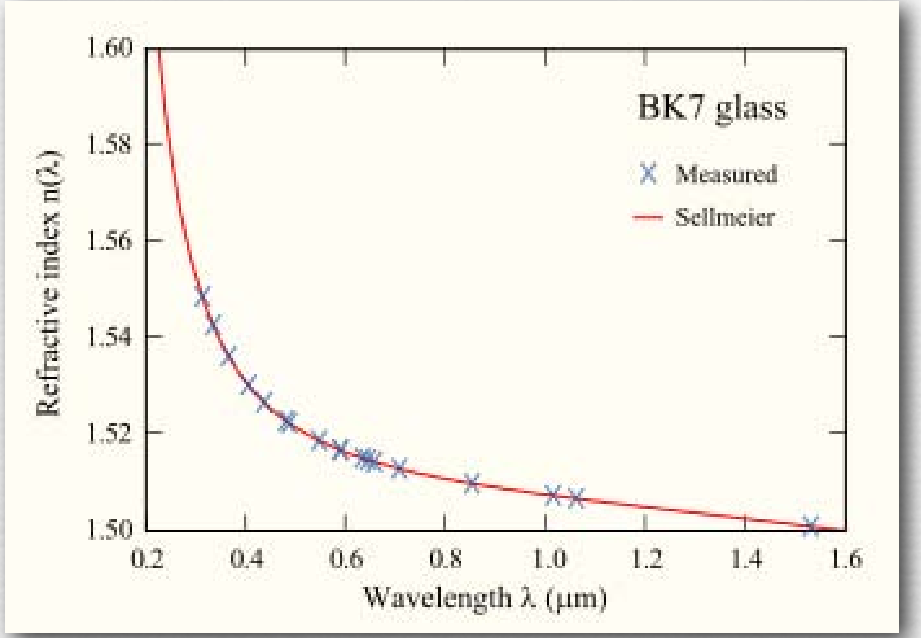
Lens aberrations

- chromatic aberrations
- Seidel aberrations, a.k.a. 3rd order aberrations
  - arise because we use spherical lenses instead of hyperbolic
  - can be modeled by adding 3rd order terms to Taylor series

$$\sin\phi \quad \approx \quad \phi \left(-\frac{\phi^3}{3!} + \frac{\phi^5}{5!} - \frac{\phi^7}{7!} + \ldots \right)$$

  - oblique aberrations
  - field curvature
  - distortion

Dispersion

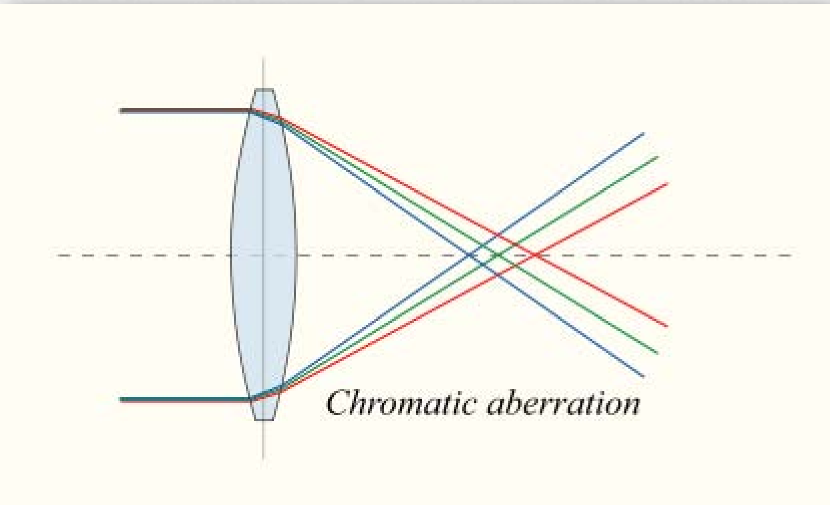


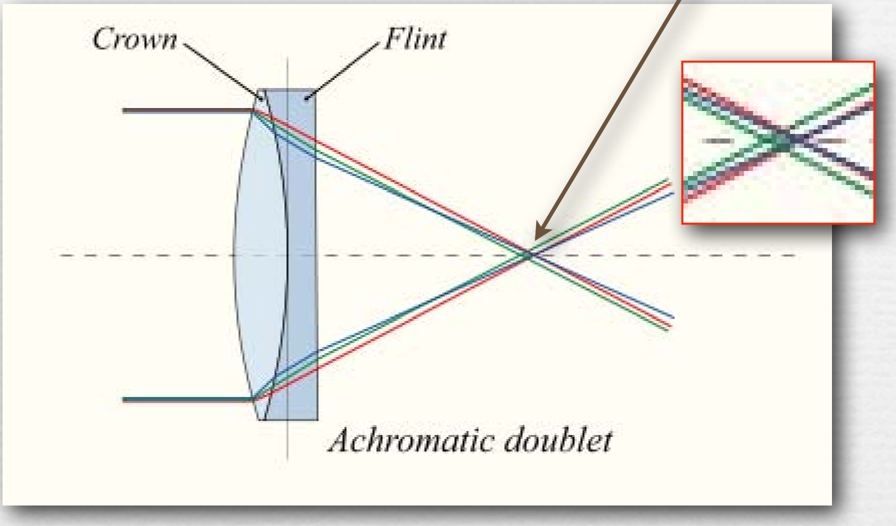
(wikipedia)

- index of refraction varies with wavelength
  - higher dispersion means more variation
  - amount of variation depends on material
  - index is typically higher for blue than red
  - so blue light bends more

Chromatic aberration

red and blue have the same focal length

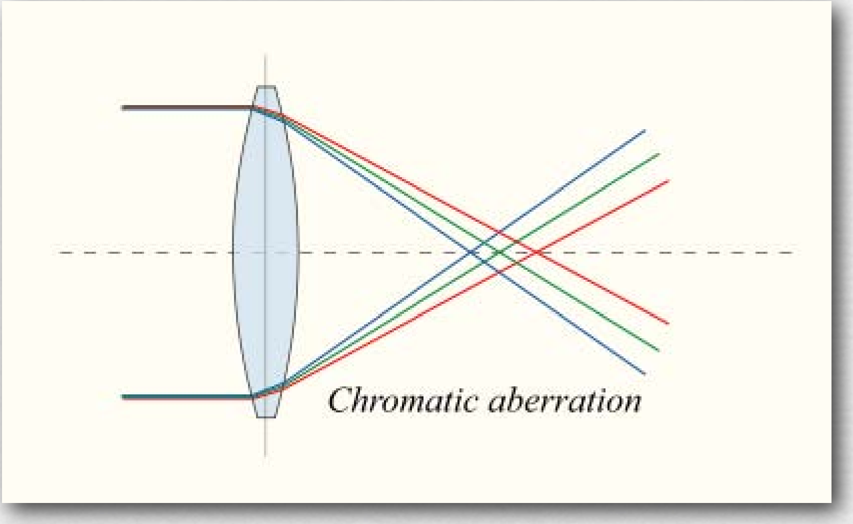


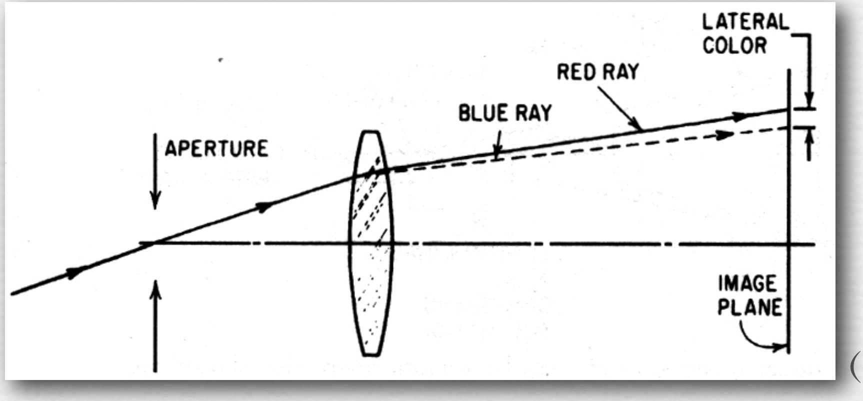


(wikipedia)

- dispersion causes focal length to vary with wavelength
  - for convex lens, blue focal length is shorter
- correct using *achromatic doublet*
  - strong positive lens + weak negative lens = weak positive compound lens
  - by adjusting dispersions, can correct at two wavelengths

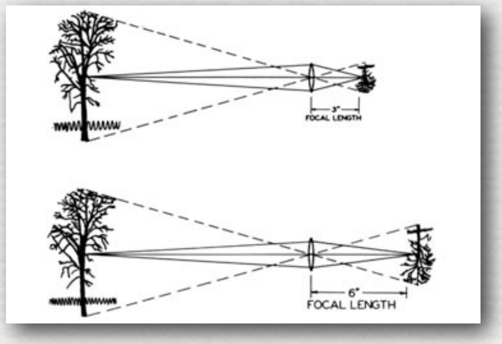
The chromatic aberrations





(Smith)

- *longitudinal (axial) chromatic aberration*
  - different colors focus at different depths, creating colorful bokehs
  - appears everywhere in the image
- *lateral (transverse) chromatic aberration*
  - if blue image is closer to lens, it will also be smaller
  - only appears at edges of images, not in the center



- can reduce longitudinal by closing down the aperture

Examples

- correctable in software
- not easily correctable

(wikipedia)

lateral

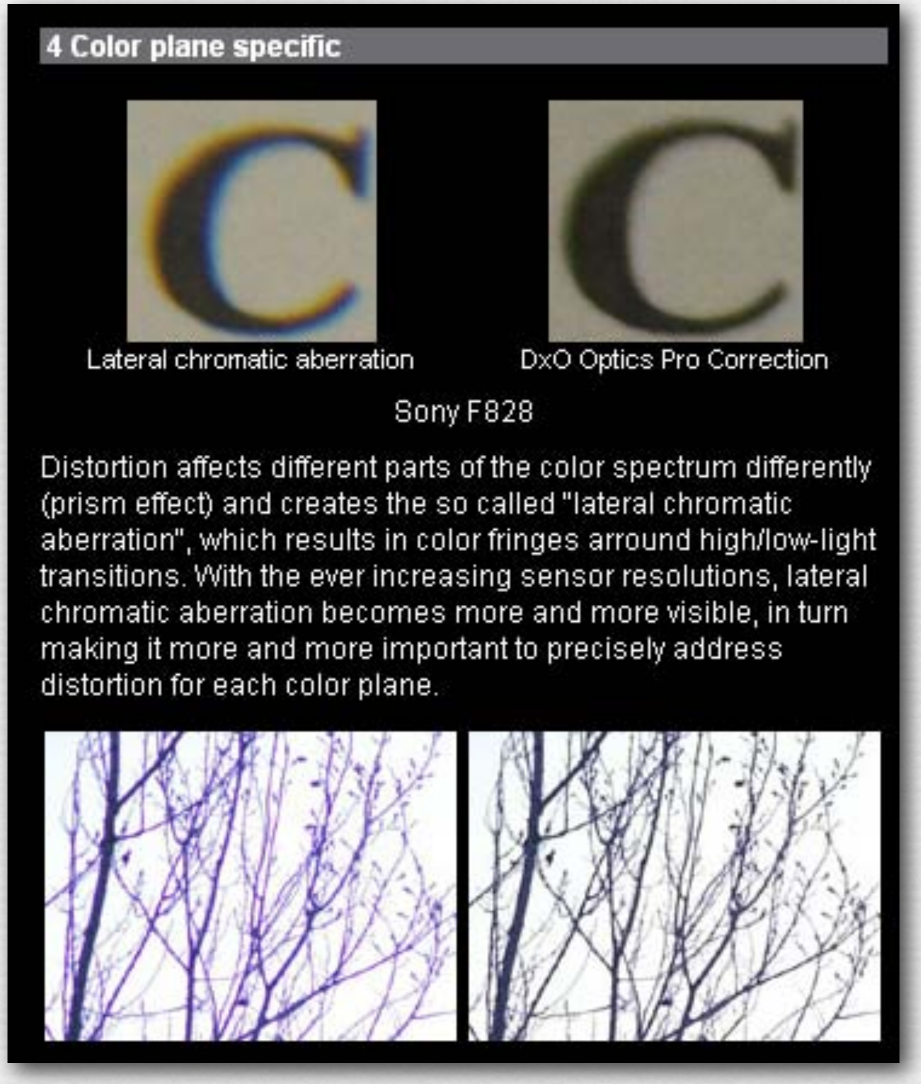
(cropped from edge of image)

(toothwalker.org)

longitudinal

- ✦ other possible causes of color fringing
  - demosiacing algorithm
  - per-pixel microlenses
  - lens flare

Software correction of lateral chromatic aberration



4 Color plane specific

Lateral chromatic aberration

DxO Optics Pro Correction

Sony F828

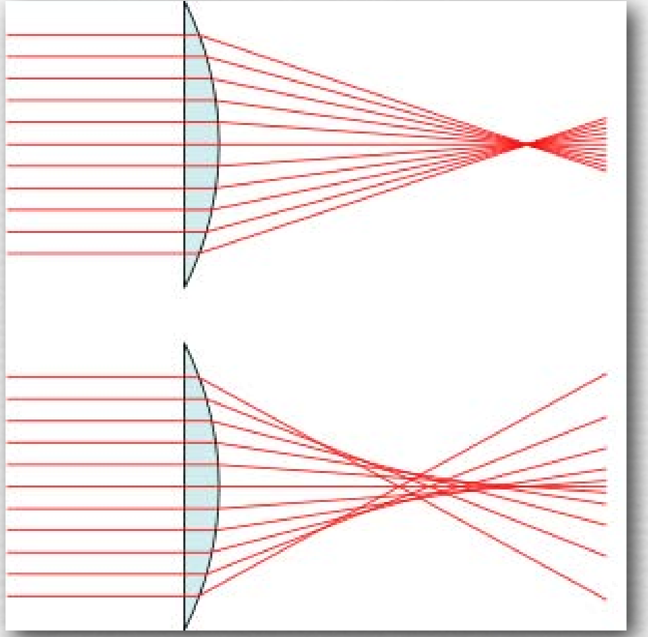
Distortion affects different parts of the color spectrum differently (prism effect) and creates the so called "lateral chromatic aberration", which results in color fringes arround high/low-light transitions. With the ever increasing sensor resolutions, lateral chromatic aberration becomes more and more visible, in turn making it more and more important to precisely address distortion for each color plane.

- ✦ Panasonic GX1 corrects for chromatic aberration in the camera
  - need focal length of lens, and focus setting
  - Adobe Camera Raw applies the same correction (Photoshop or Lightroom)

Q. Why don't humans see chromatic aberration?

Spherical aberration

hyperbolic lens



spherical lens

(wikipedia)

- focus varies with ray height (distance from optical axis)
- can reduce by stopping down the aperture
- can correct using an aspherical lens
- can correct for this and chromatic aberration
  by combining with a concave lens of different properties

Examples


(Canon)

sharp

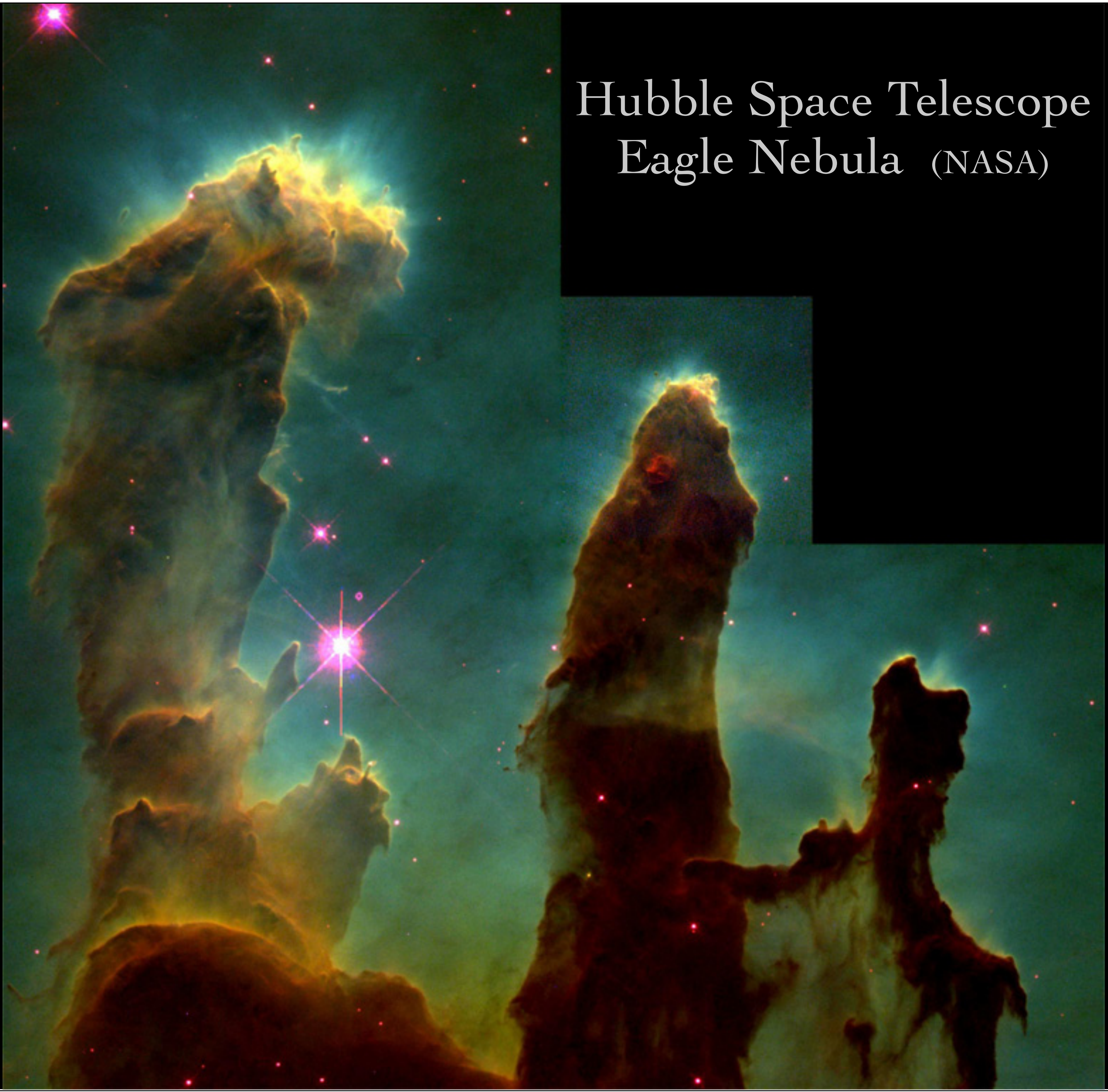
soft focus

Canon 135mm f/2.8 soft focus lens

Hubble telescope

before correction

after correction



Hubble Space Telescope
Eagle Nebula (NASA)

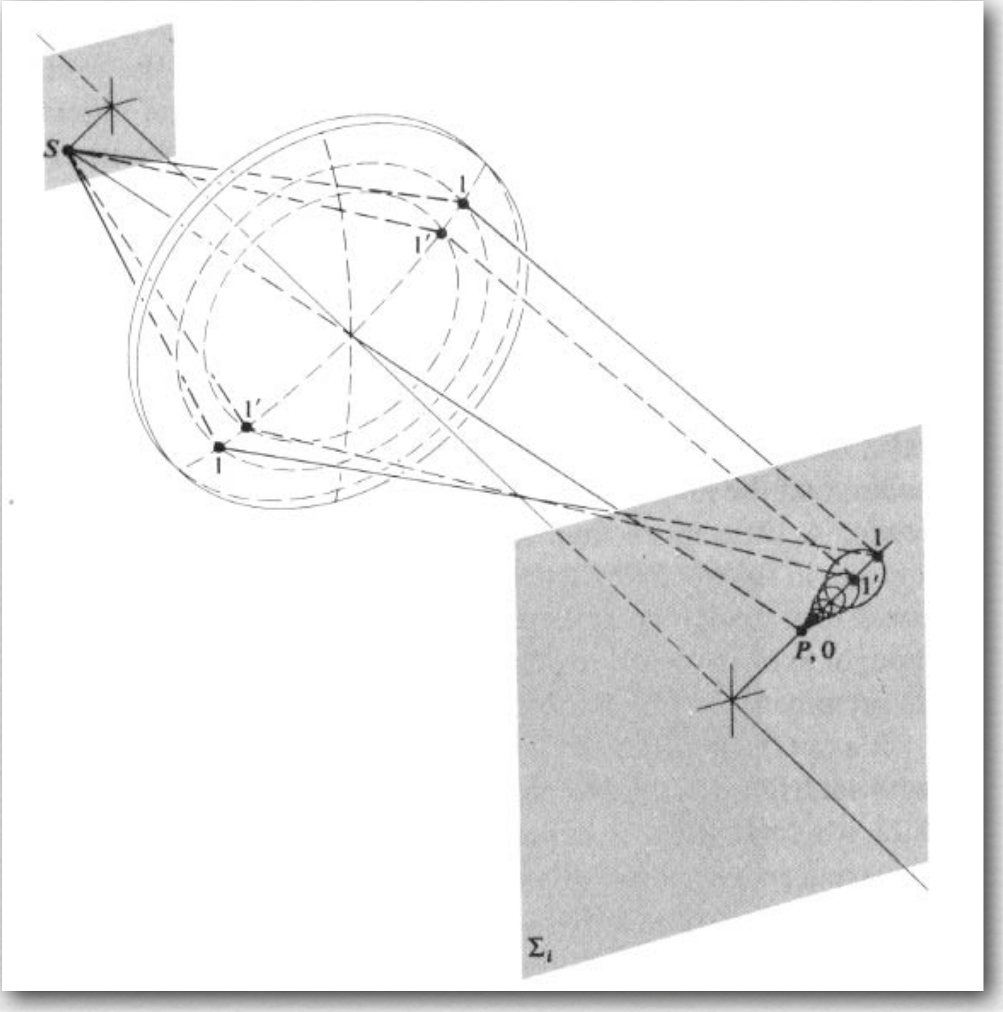
Aberrations and their properties

- lateral chromatic aberrations do not appear in center of field
  - they get worse with increasing distance from the optical axis
  - cannot reduce by closing down the aperture

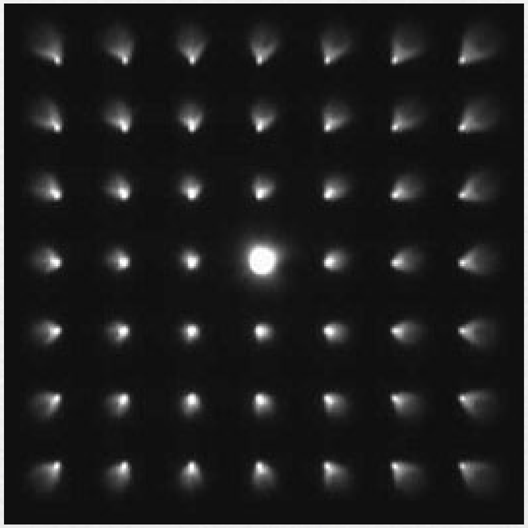
- longitudinal chromatic & spherical aberrations occur everywhere in the field of view
  - on and off the optical axis
  - can reduce by closing down the aperture

- oblique aberrations do not appear in center of field
  - they get worse with increasing distance from the optical axis
  - can reduce by closing down the aperture
  - coma and astigmatism

Coma



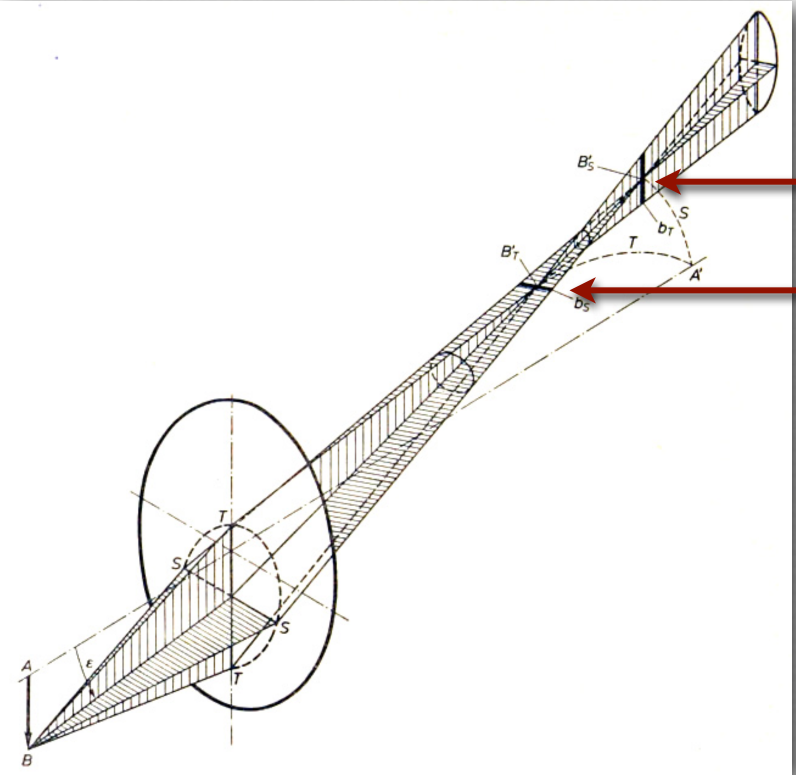
(Hecht)



(ryokosha.com)

- magnification varies with ray height (distance from optical axis)

Astigmatism



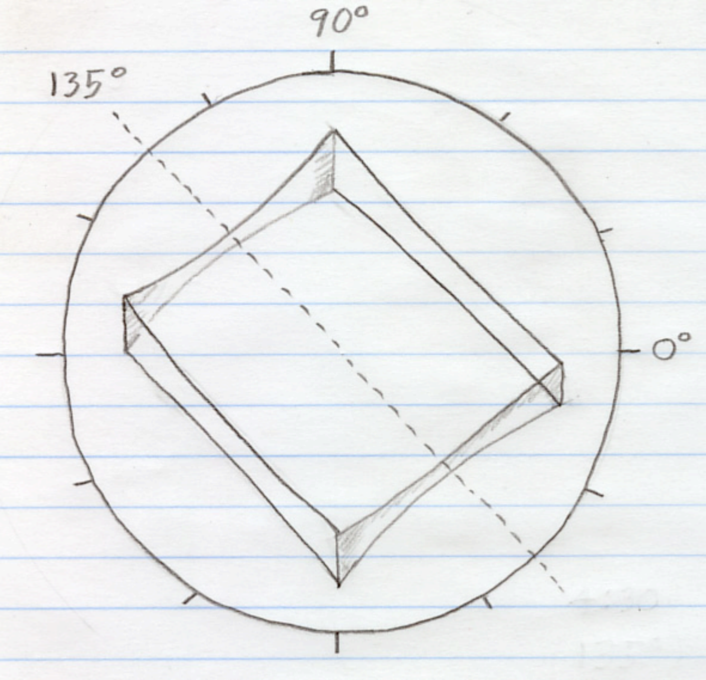
focus of sagittal rays

focus of transverse rays

(Pluta)

- transverse and sagittal rays focus at different depths
- my full eyeglass prescription
  - right: -0.75 -1.00 axis 135, left: -1.00 -0.75 axis 180

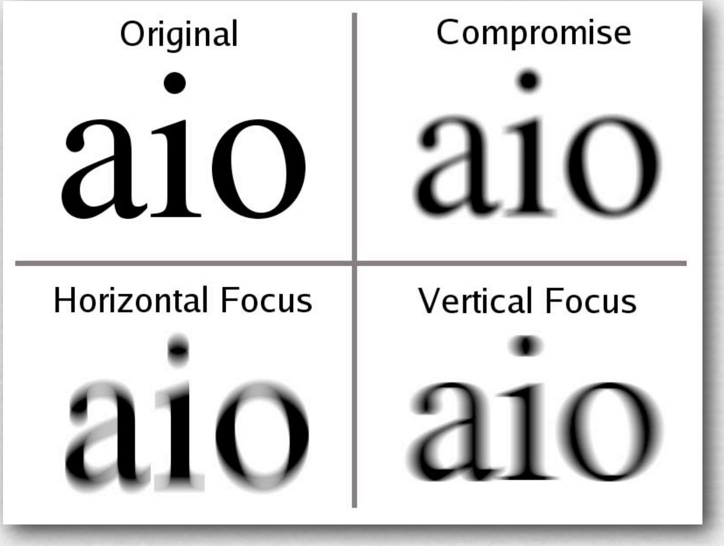
Correcting astigmatism using a cylindrical lens (contents of whiteboard)



- for myopia + astigmatism, one needs a spherical lens + cylindrical lens, i.e. a lens with different radii of curvature in two perpendicular directions
    - in my right eye, first direction has focal length -1 /0.75 = -1.33 meters, and second direction has focal length -1 / 1.00 = -1.00 meters
- lens is then rotated around the optical axis before mounting in frame
    - in my case extrusion axis of second curvature is 135º (10:30 - 4:30 on the clock)

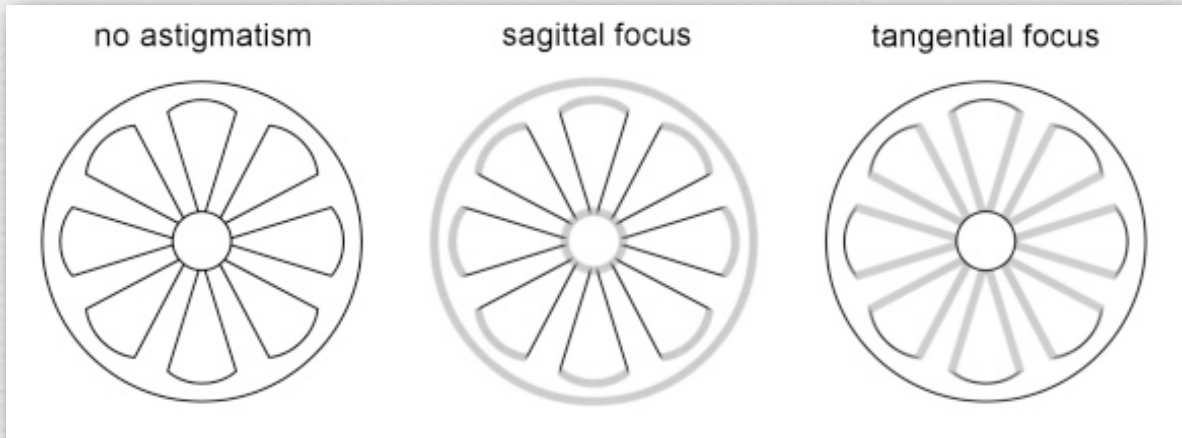
Two kinds of astigmatism

(Wikipedia)



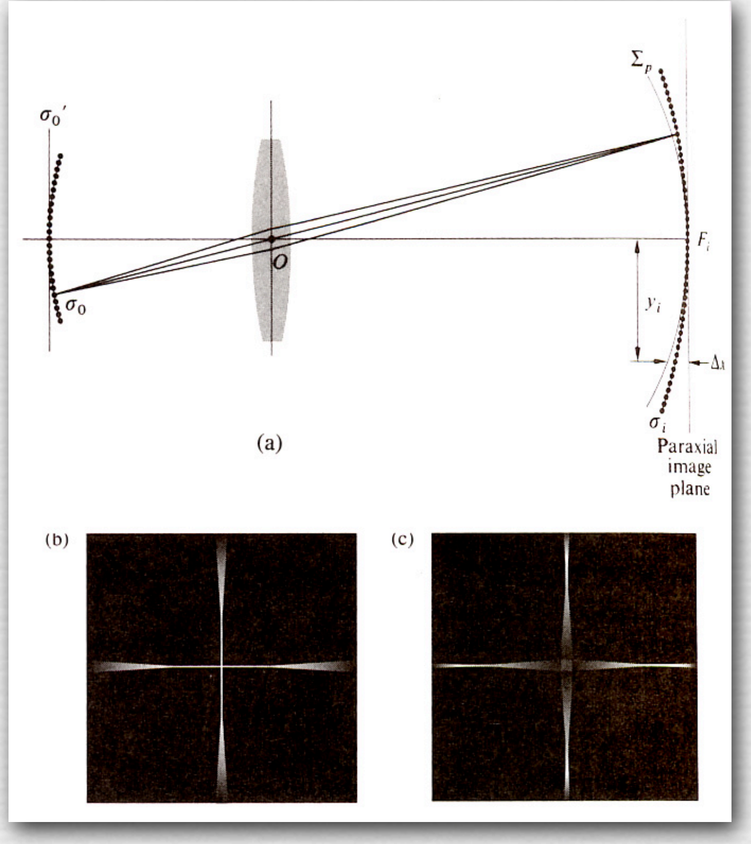
ophthalmic astigmatism
(due to oblong eye)

(http://toothwalker.org/optics/astigmatism.html)



optical astigmatism
(even in rotationally symmetric photographic lenses)

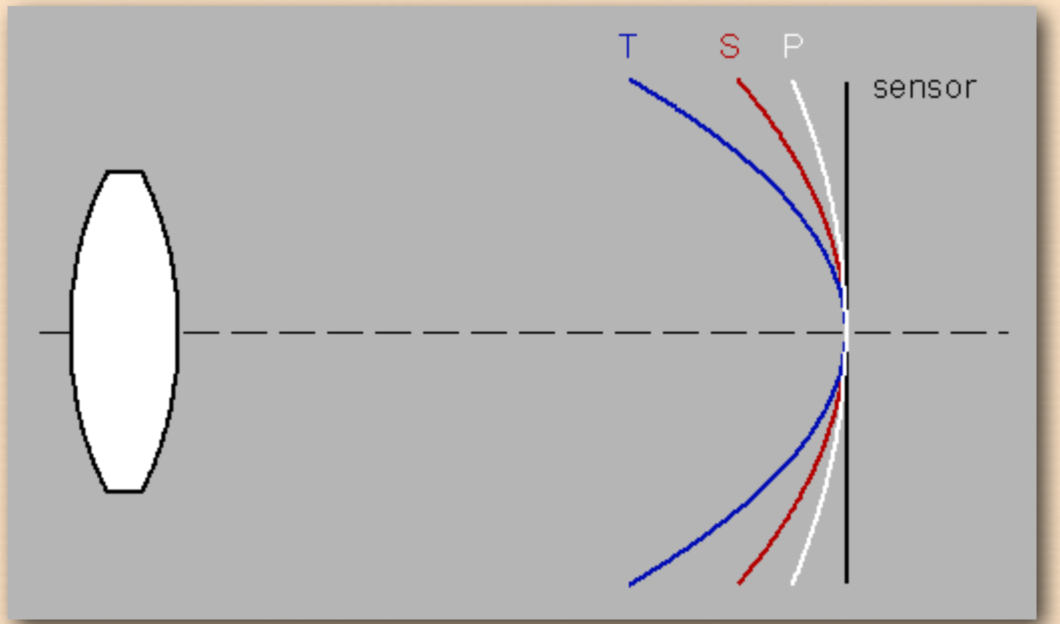
Field curvature



(Hecht)

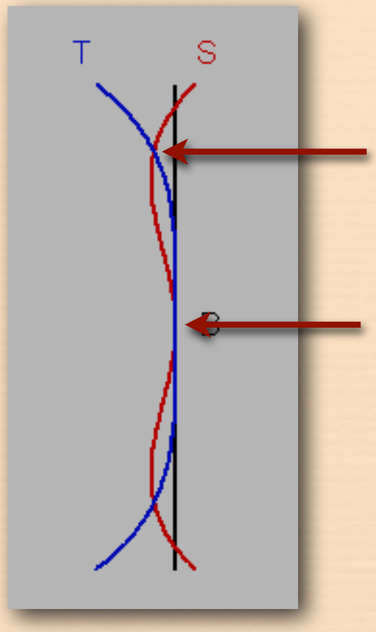
- spherical lenses focus a curved surface in object space onto a curved surface in image space
- so a plane in object space cannot be everywhere in focus when imaged by a planar sensor

Correcting for astigmatism and field curvature



uncorrected lens

(toothwalker.org)



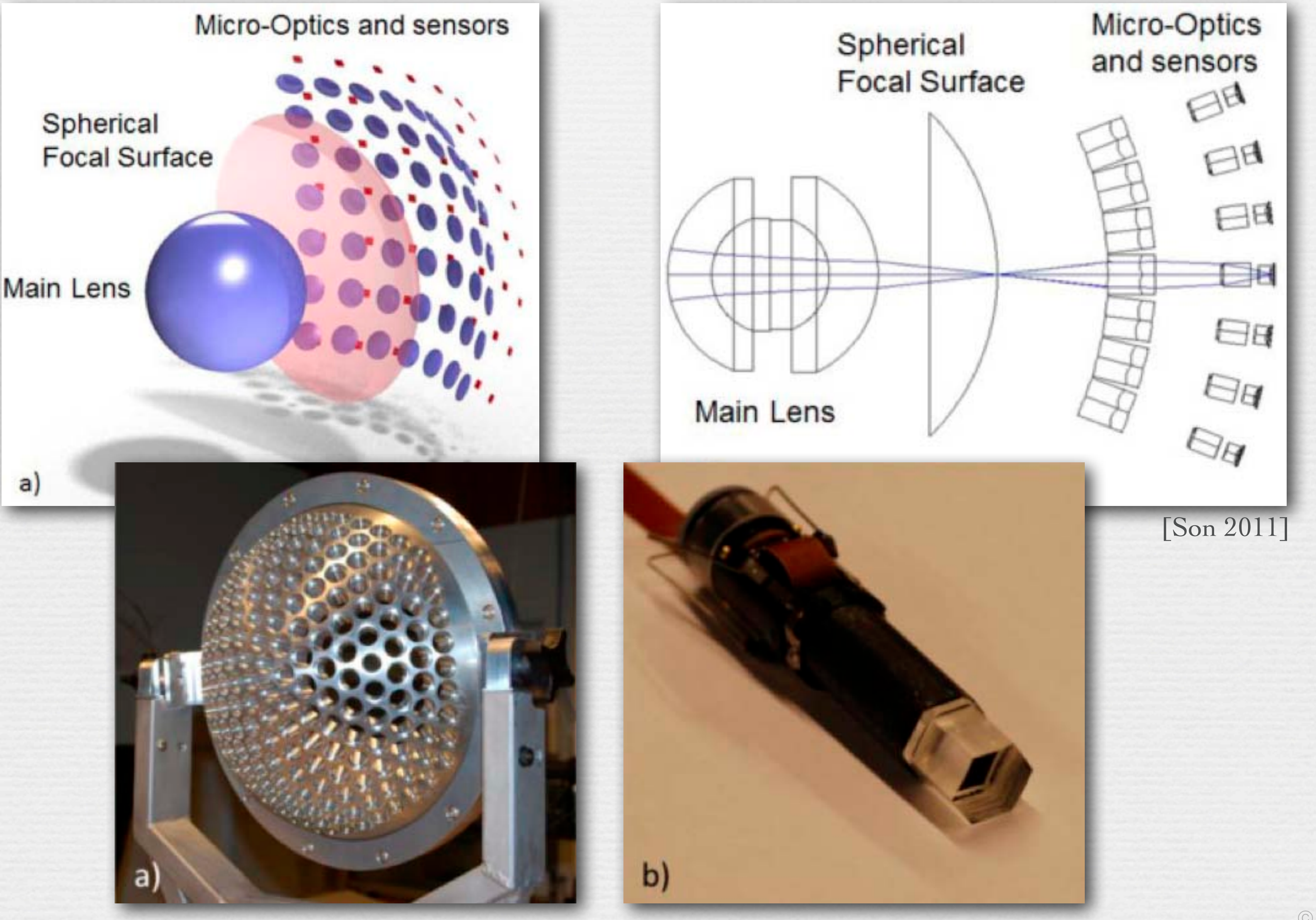
no astigmatism

no astigmatism

anastigmat

- spherical lenses image planes onto curved surfaces
  - T = tangential focus surface
  - S = sagittal focus surface
  - P = Petzval surface (field curvature only, if there were no astigmatism)
- anastigmat makes T = S at axis and one other ray height

A spherical focus surface camera

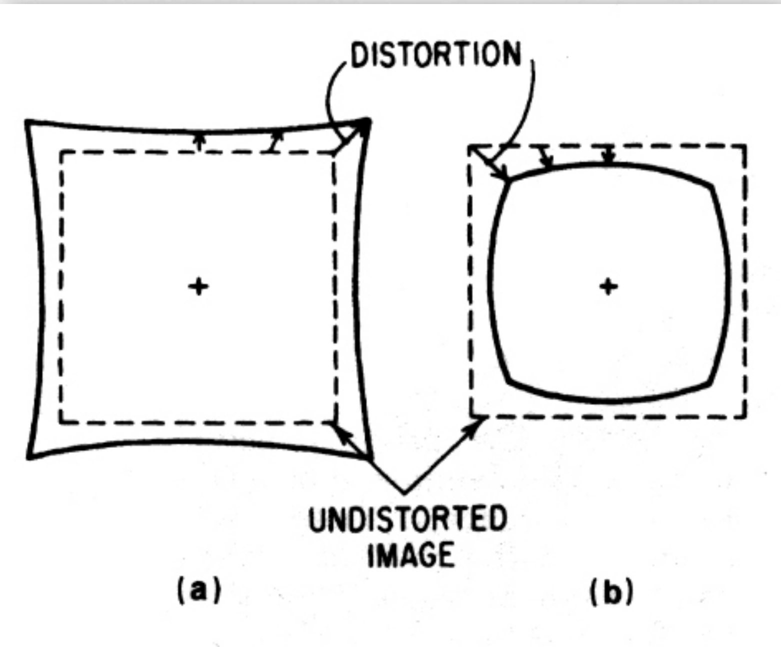


[Son 2011]

Distortion

- correctable in software

(Smith)



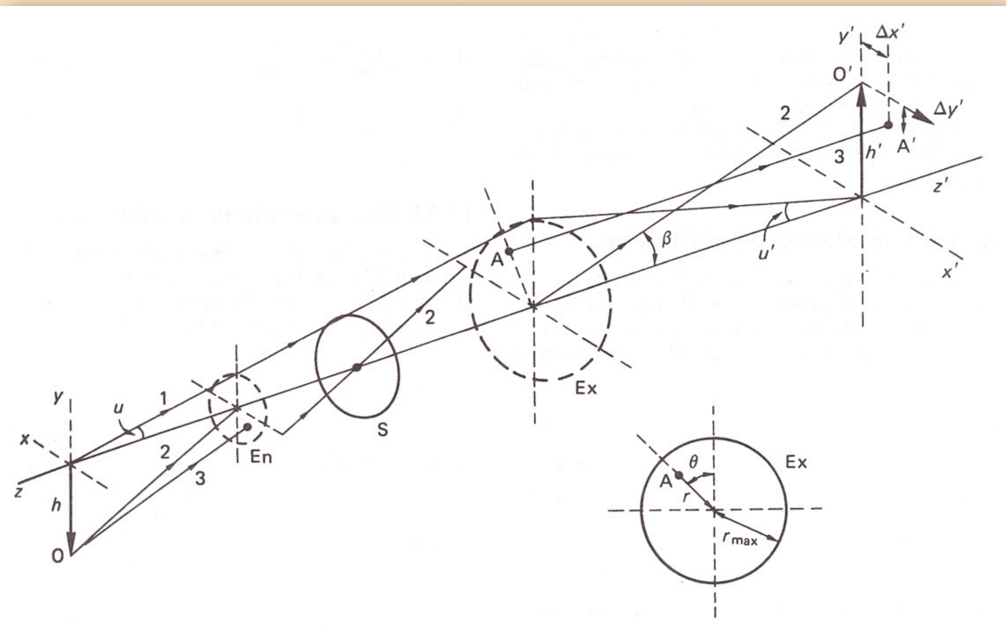
(Kingslake)

pincushion distortion

- change in magnification with image position
  - (a) pincushion
  - (b) barrel
- closing down the aperture does not improve this

Algebraic formulation of monochromatic lens aberrations

Not responsible on exams
for orange-tinted slides



(Smith)

- spherical aberration $a_s r^4$
- coma $a_c h' r^3 \cos\theta$
- astigmatism $a_a h'^2 r^2 \cos^2\theta$
- field curvature $a_d h'^2 r^2$
- distortion $a_t h'^3 r \cos\theta$

Recap

- all lenses are subject to chromatic aberration
  - longitudinal appears everywhere; lateral is worse at edges
  - only longitudinal can be reduced by closing down aperture
  - both can be partly corrected using more lenses, and lateral can be partly corrected using software

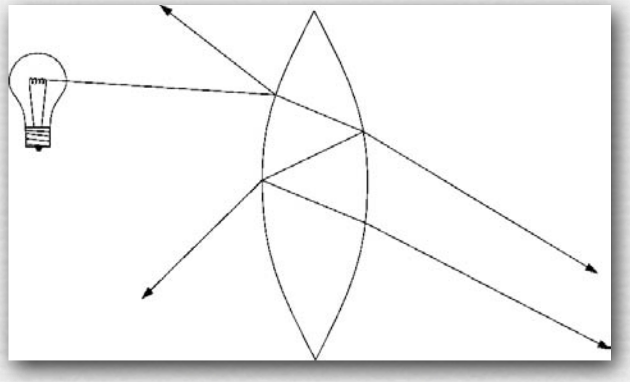
- all <u>spherical</u> lenses are subject to Seidel aberrations: spherical, coma, astigatism, field curvature, distortion
  - some appear everywhere; others only at edges
  - all but distortion can be reduced by closing down aperture
  - only distortion can be corrected completely in software

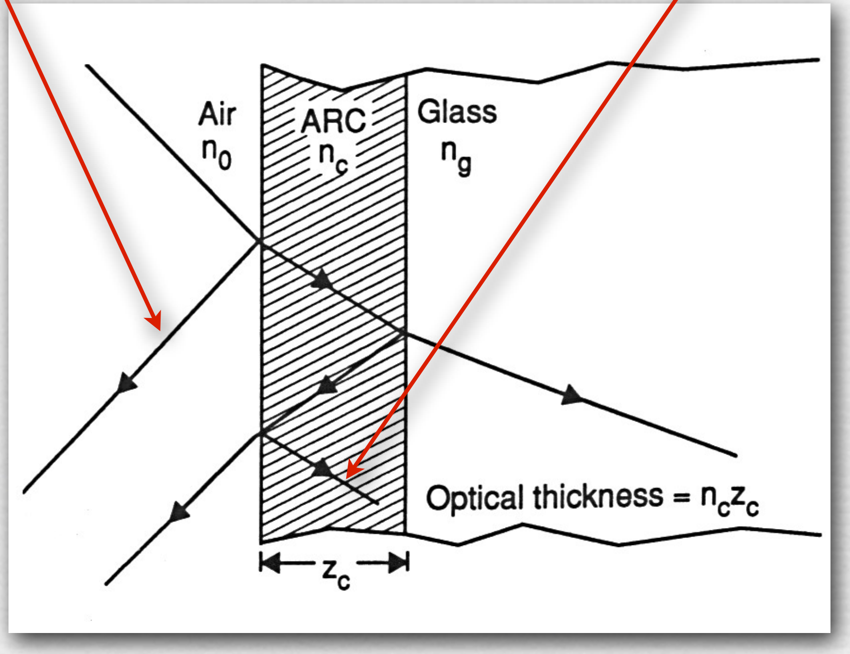
Questions?

Veiling glare

in the outermost lens we don't care about killing this reflection

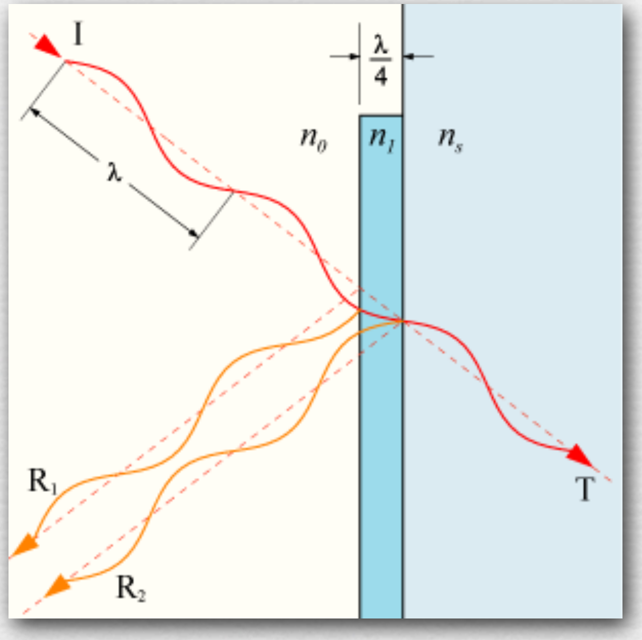
but we do care about killing this one

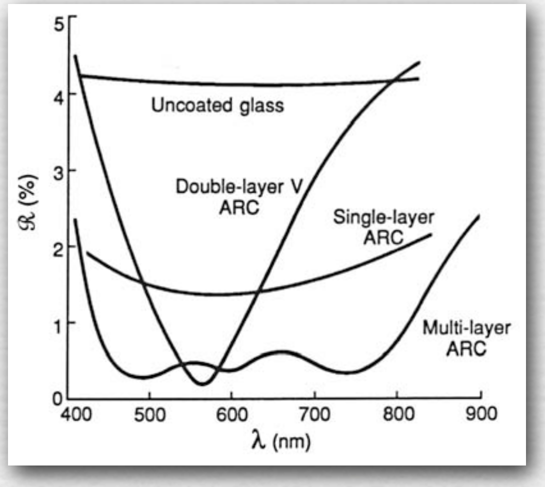




- contrast reduction caused by stray reflections
- can be reduced by anti-reflection coatings

Veiling glare

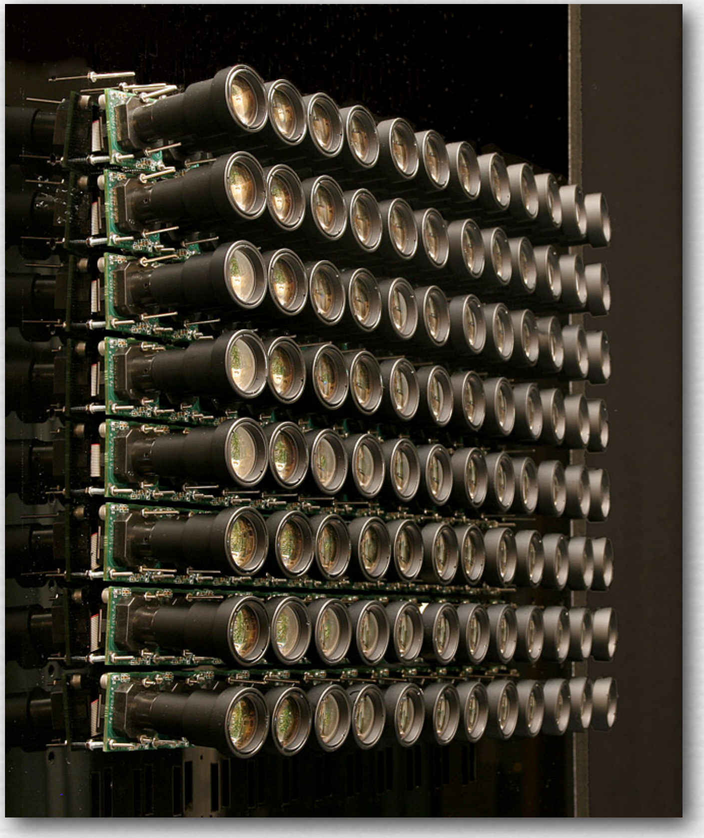

(wikipedia)



- ✦ contrast reduction caused by stray reflections
- ✦ can be reduced by anti-reflection coatings
  - based on interference, so optimized for one wavelength
  - to cover more wavelengths, use multiple coatings

Camera array with too much glare

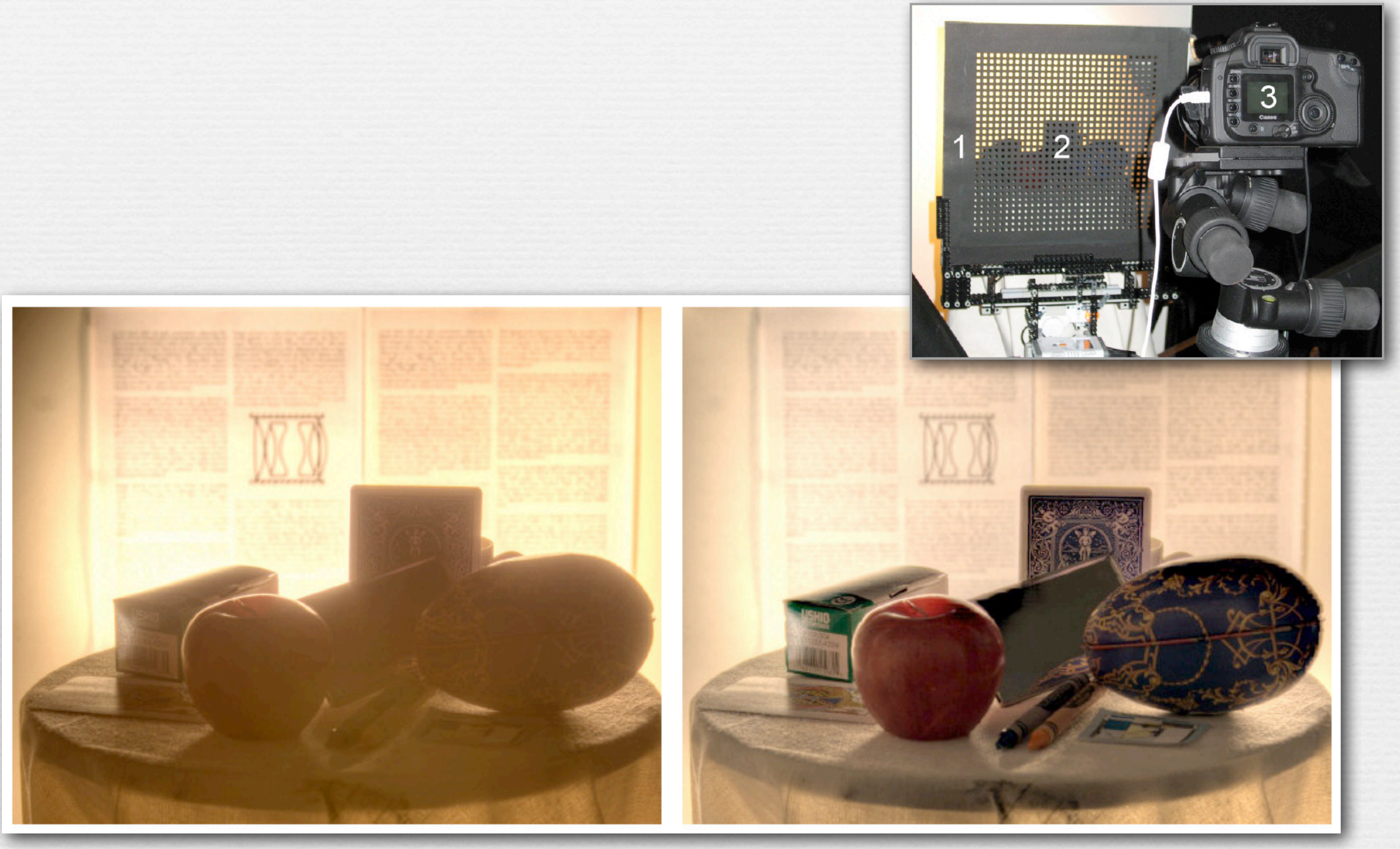
Stanford
Multi-Camera
Array



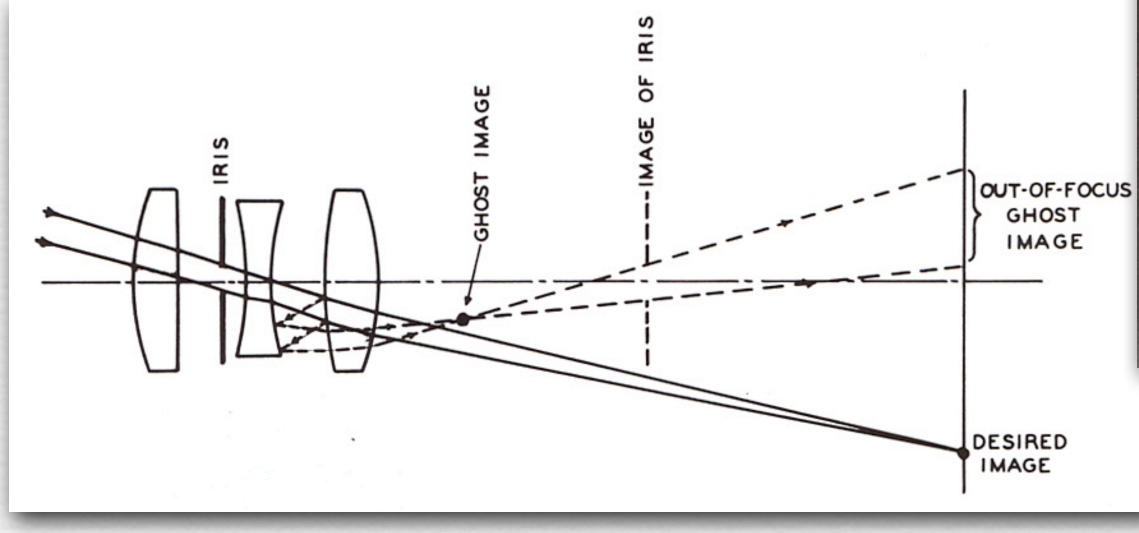
- 12 × 8 array of 600 × 800 pixel webcams = 7,200 × 6,400 pixels
- goal was highest-resolution movie camera in the world
- failed because glare in inexpensive lenses led to poor contrast

Removing veiling glare computationally

[Talvala, Proc. SIGGRAPH 2007]



Flare and ghost images

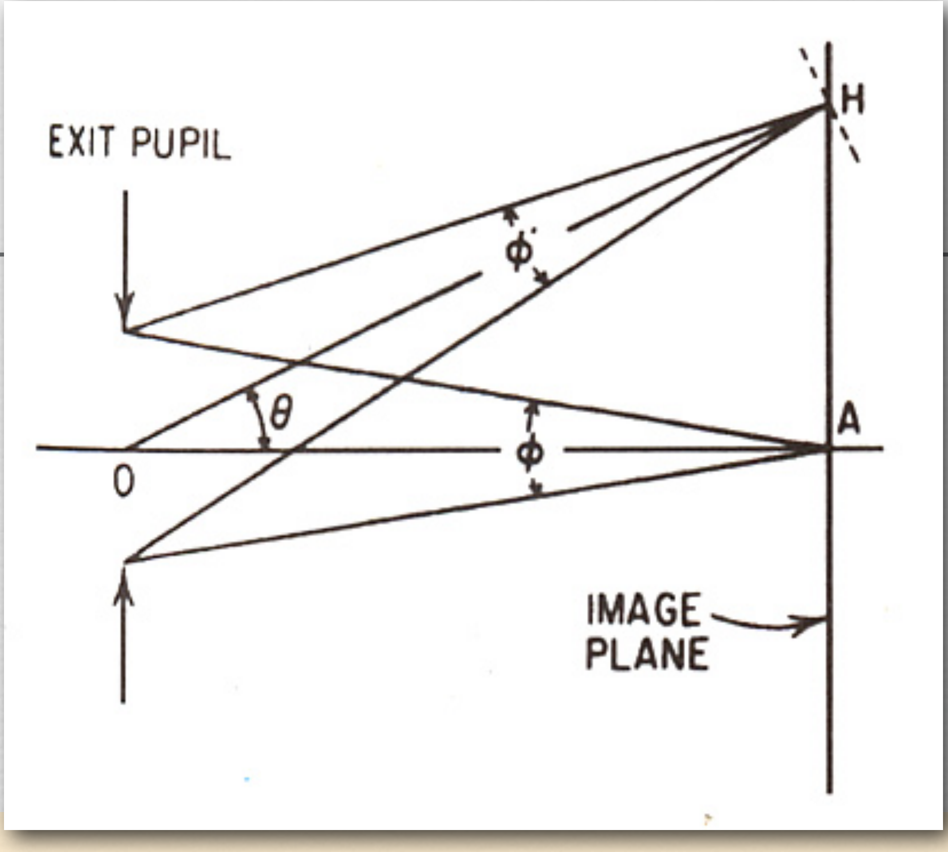


(Kingslake)

- reflections of the aperture, lens boundaries, etc., i.e. things inside the camera body
- removing these artifacts is an active area of research in computational photography
- but it's a hard problem

Vignetting

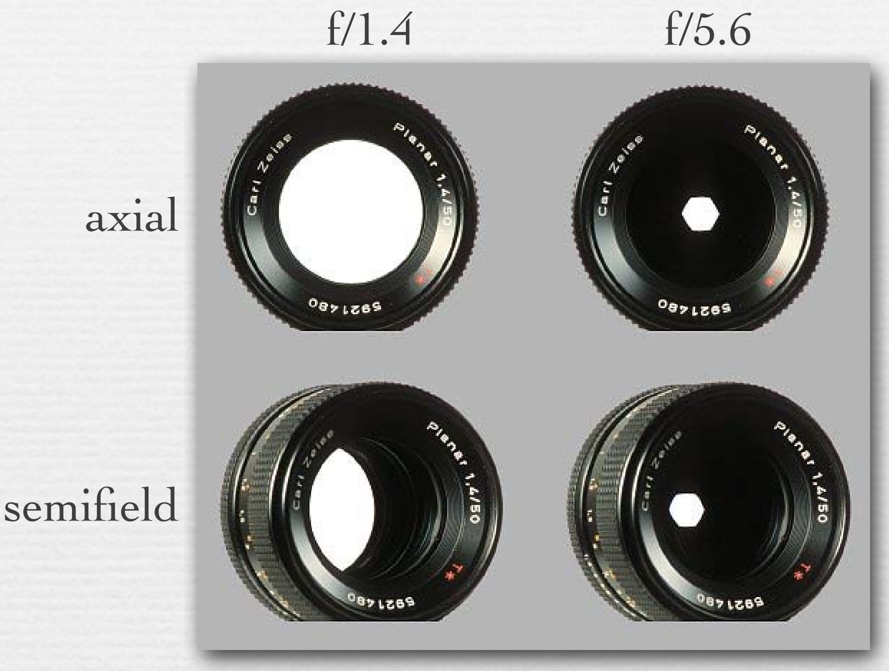
(a.k.a. **natural** vignetting)



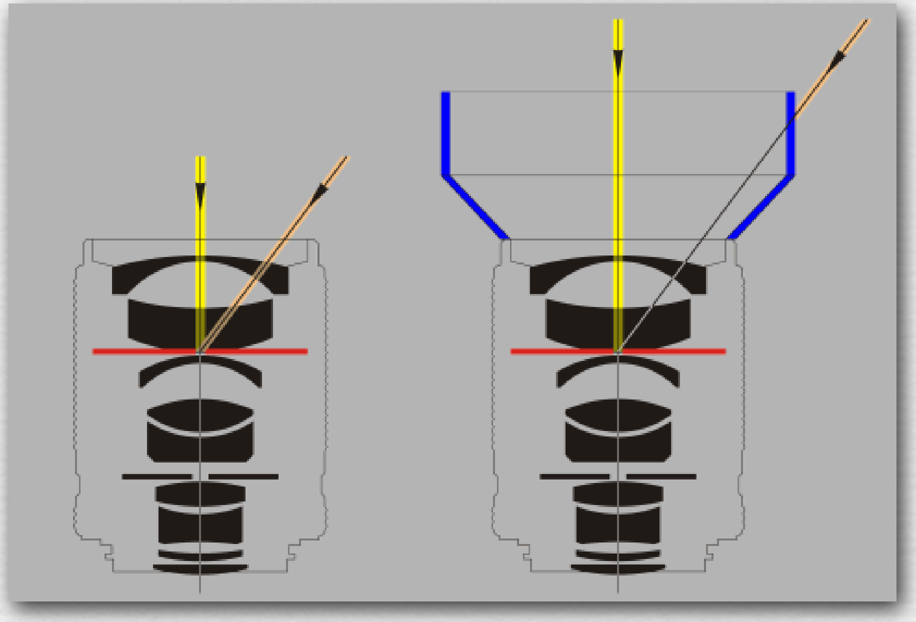
(Smith)

- irradiance is proportional to projected area of aperture as seen from pixel on sensor, which drops as cos θ
- irradiance is proportional to projected area of pixel as seen from aperture, which also drops as cos θ
- irradiance is proportional to distance^2 from aperture to pixel, which rises as 1/cos θ
- combining all these effects, light drops as $\cos^4 \theta$

Other sources of vignetting



optical vignetting
from multiple lens elements,
especially at wide apertures



(toothwalker.org)

mechanical vignetting
from add-on lens hoods
(or filters or fingers)

- **pixel** vignetting due to shadowing inside each pixel
(we'll come back to this)

Examples

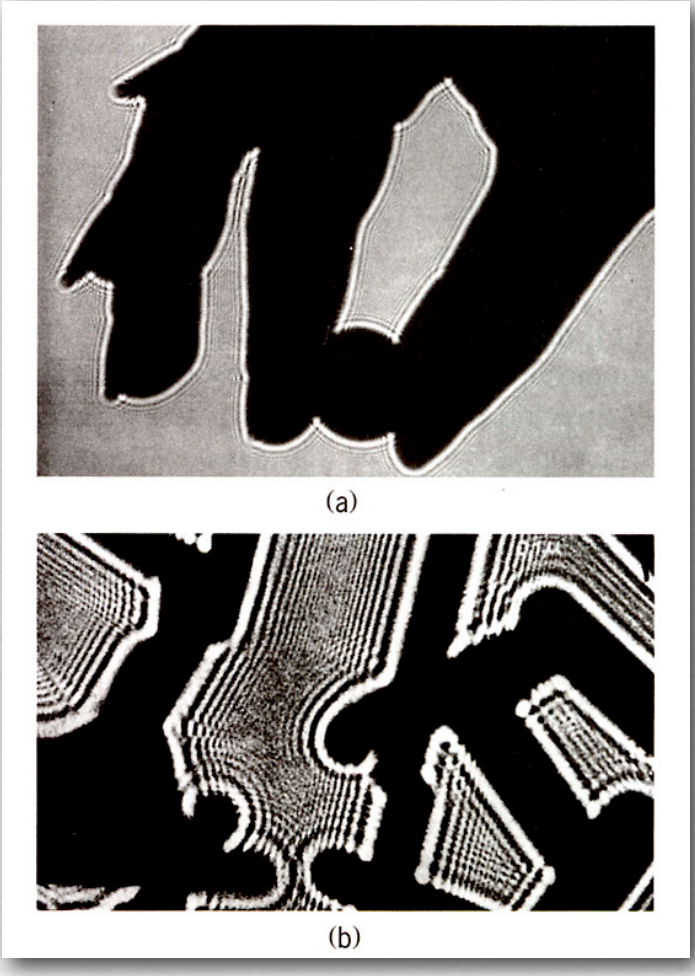
(toothwalker.org)

(toothwalker.org)

(wikipedia)

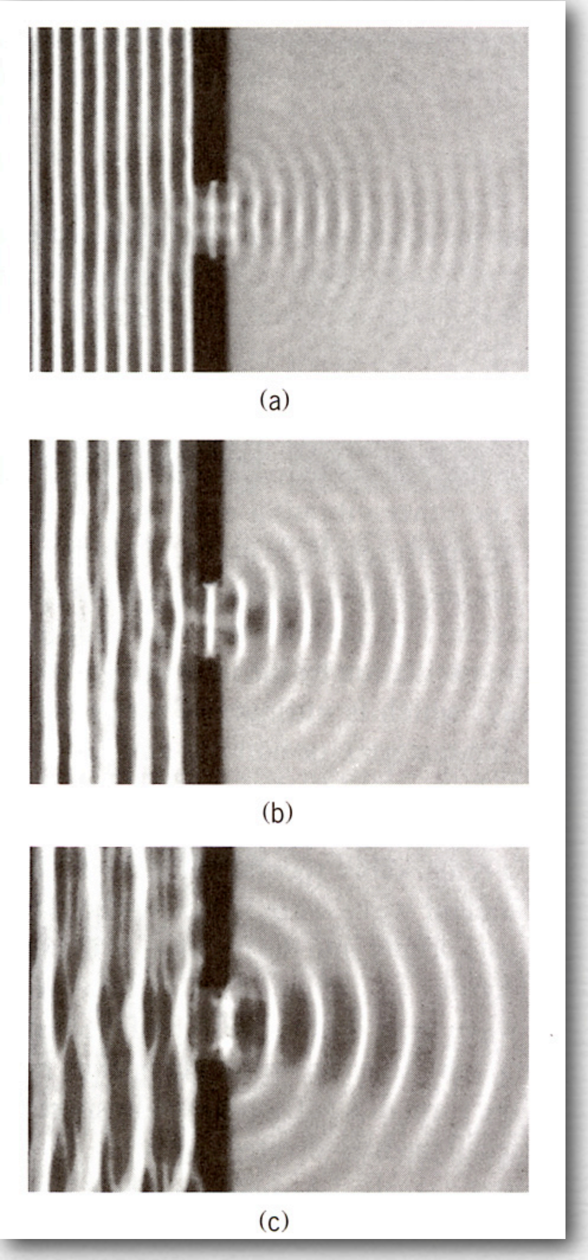
- vignetting causes falloff in brightness towards edges of image
- vignetting affects the *bokeh* of out-of-focus features
- vignetting is correctable in software (except for bokeh effects), but boosting pixel values worsens noise
- vignetting can be applied afterwards, for artistic effects

Diffraction



(Hecht)

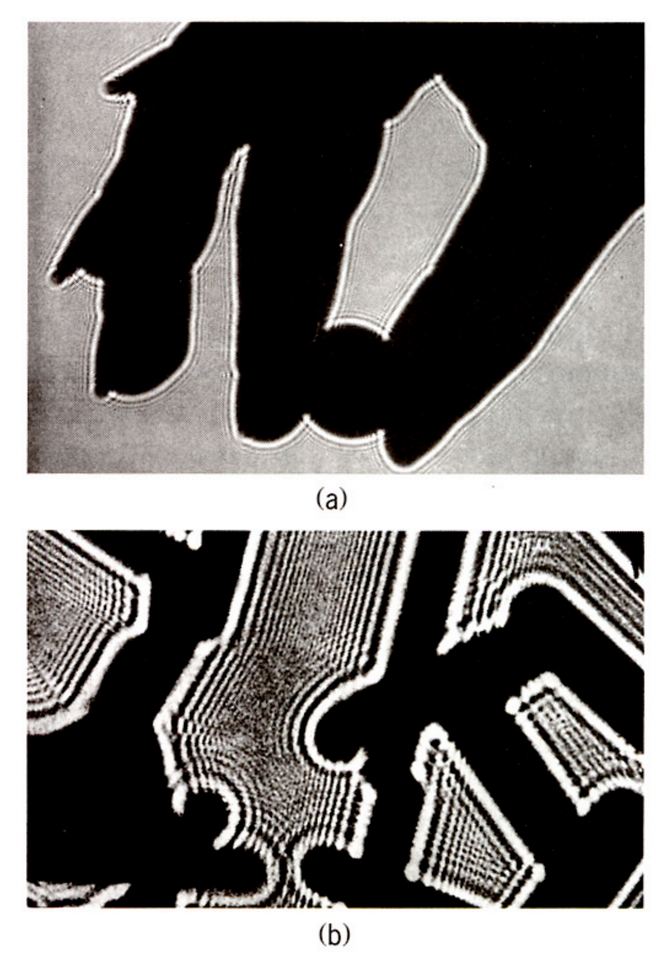
illuminated by a
(spread-out) laser beam
& recorded directly on film



varying the wavelength
of waves passing through
a slit in a ripple tank

- as wavelength decreases in the ripple tank (c⟶a), propagation becomes more ray-like

Diffraction



(Hecht)

illuminated by a
(spread-out) laser beam
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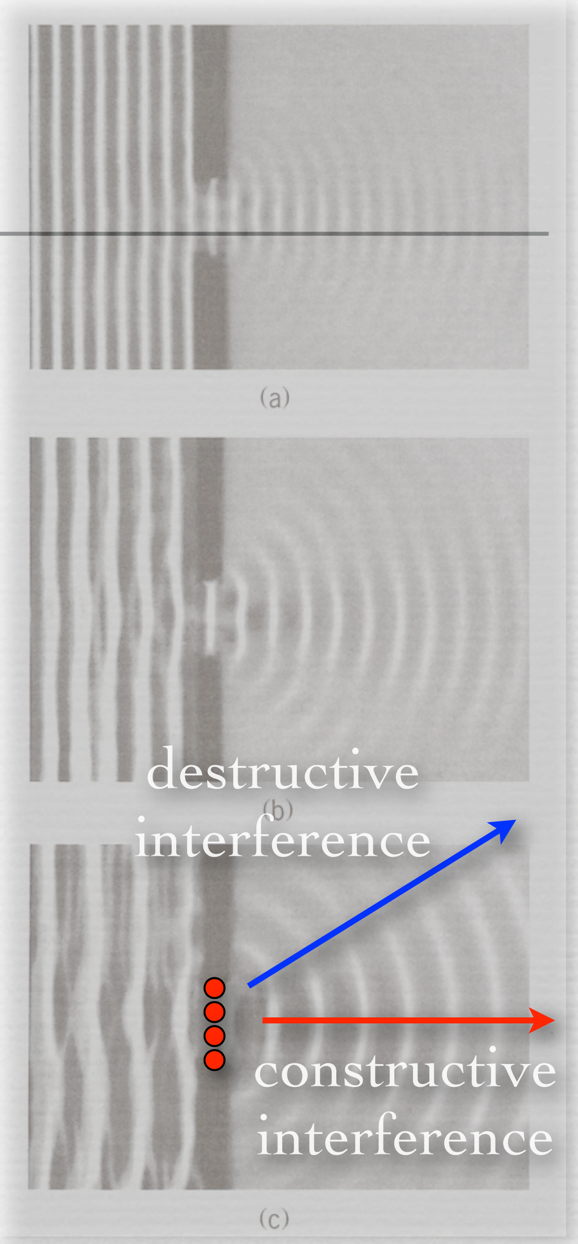
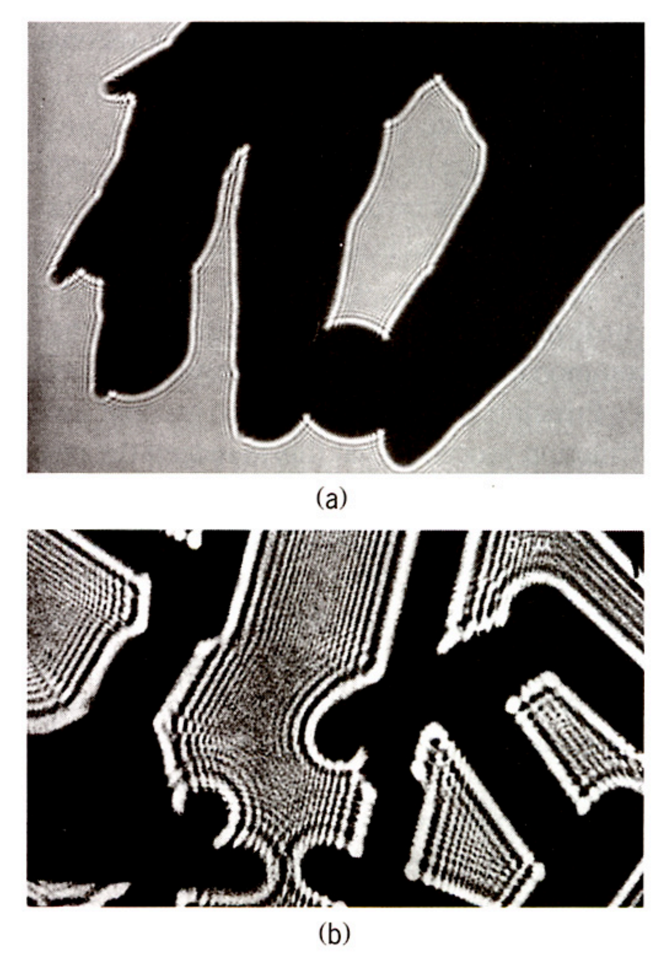


destructive interference

constructive interference

- as wavelength decreases in the ripple tank (c⟶a), propagation becomes more ray-like

Diffraction



(Hecht)

illuminated by a
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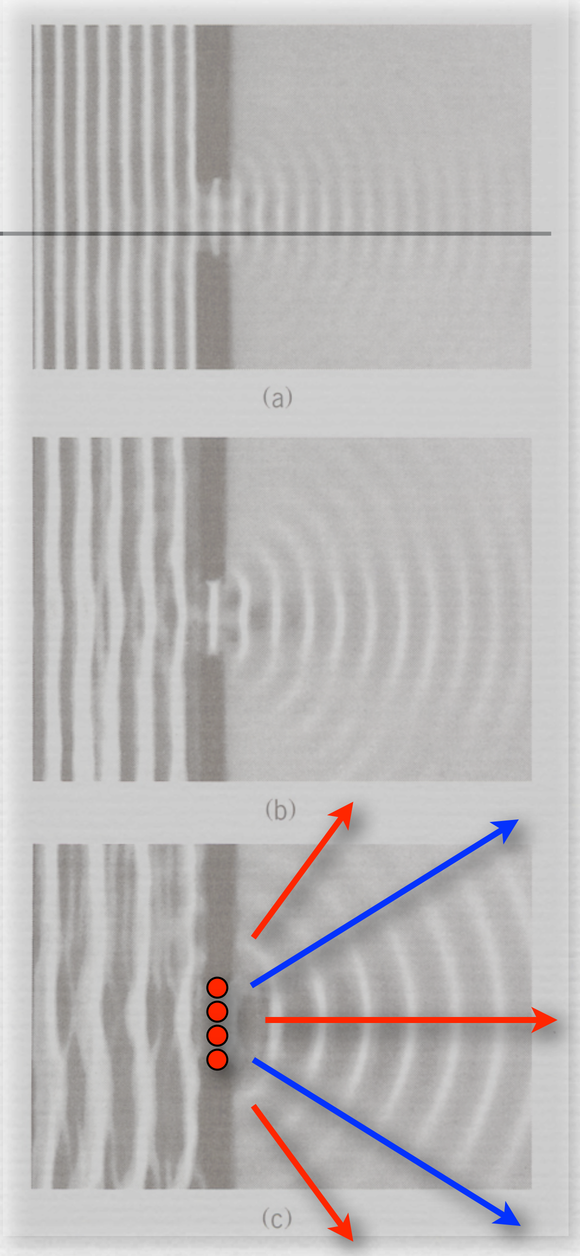

varying the wavelength
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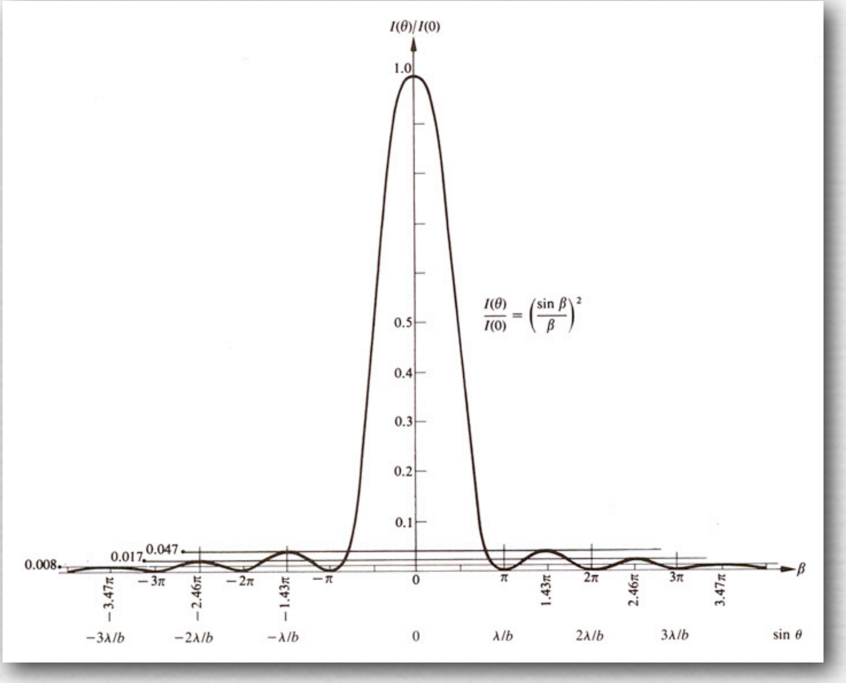
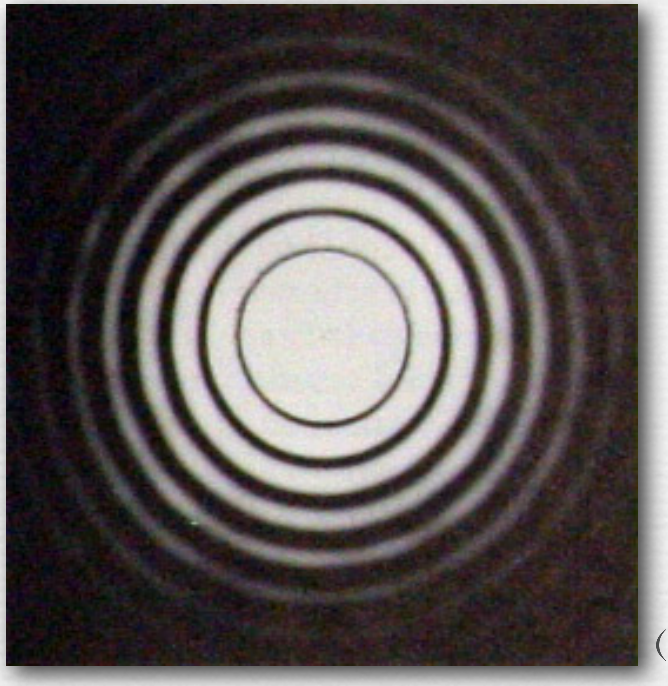
Airy rings



diffraction from a slit



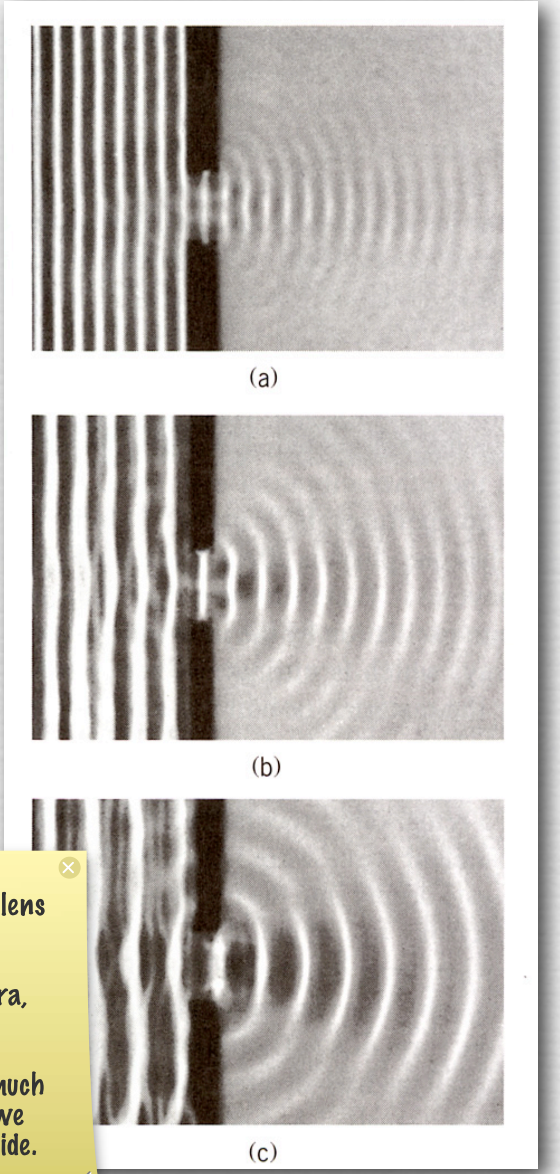
(Hecht)

diffraction from a circular aperture: Airy rings

- if the illumination were a laser, a lens would produce this pattern
- but considering all wavelengths, the dark rings vanish, leaving a blur

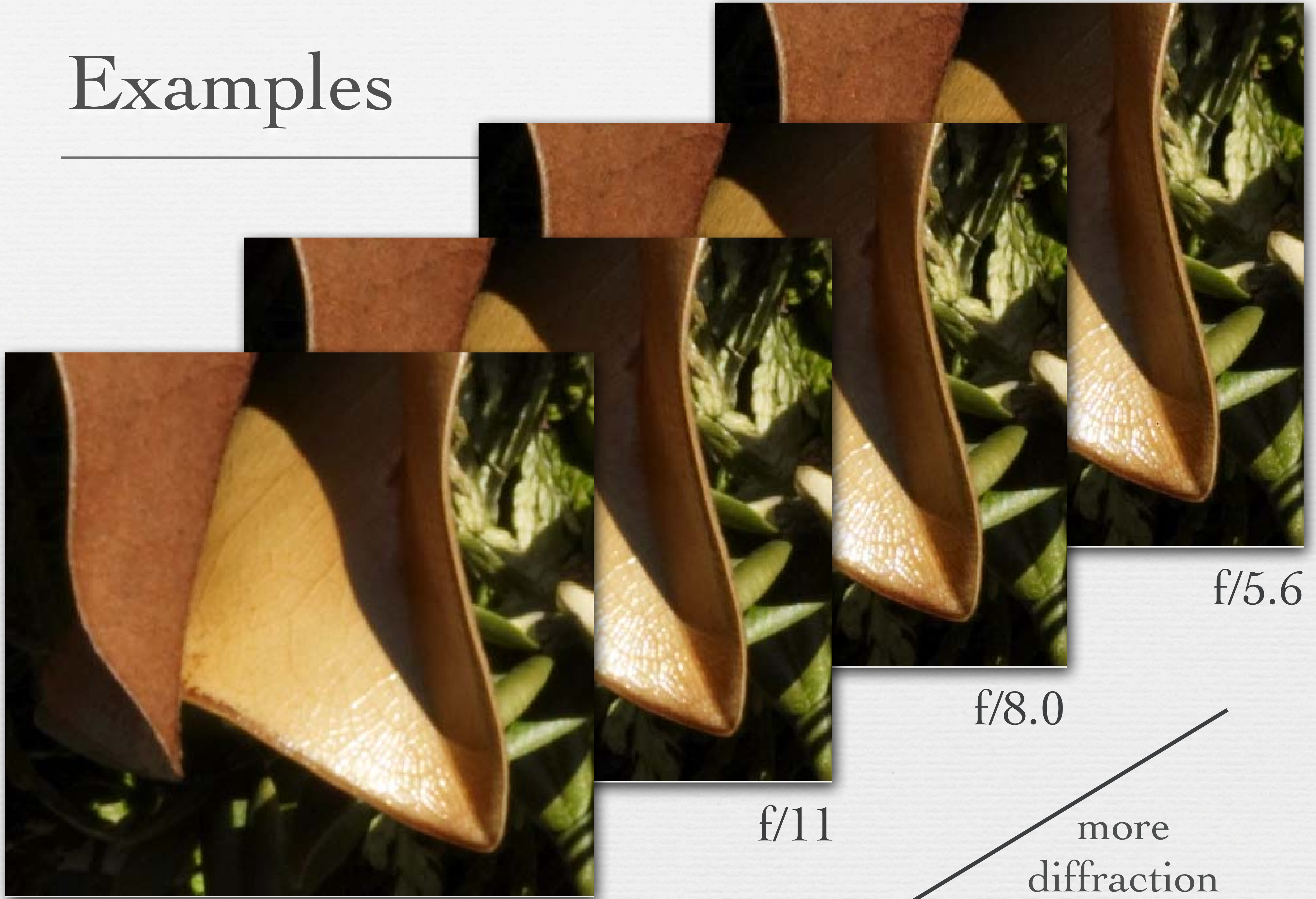
Diffraction in photographic cameras

- well-corrected lenses are called *diffraction-limited*
- the smaller the aperture (A) (or the longer the wavelength), the larger the diffraction blur
- the longer the distance to the sensor (f), the larger the blur when it gets there
- thus, the size of the blur varies with $N = f / A$



A student noted that the distance between lens and sensor in a camera depends not only on focal length f, but also on image distance s_i. Thus, it would change when focusing a camera, not only when zooming it. This is true. However the change in lens-sensor distance when focusing a camera is very small, and much smaller than the change when zooming, so we ignore it in the rule of thumb given on this slide.

Examples



f/5.6

f/8.0

f/11

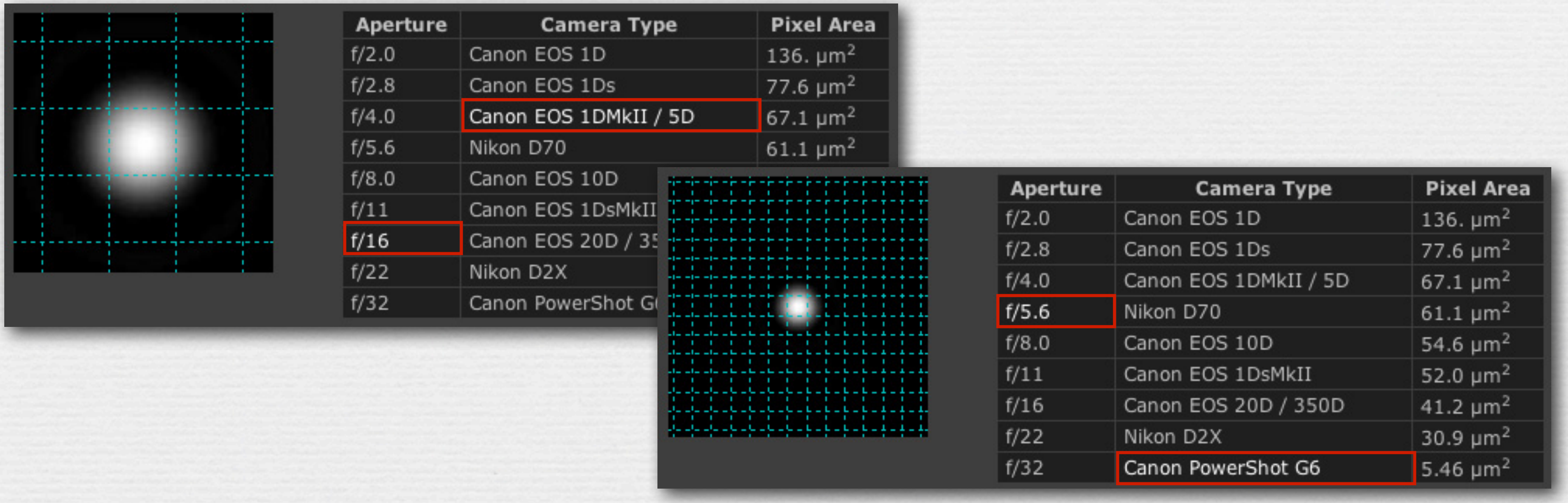
f/22

more
diffraction

(luminous-landscape.com)

Diffraction in photographic cameras

- the smaller the pixels, the more of them the blur covers
  - if the blur spans >> 1 pixel, we begin to complain



Aperture	Camera Type	Pixel Area
f/2.0	Canon EOS 1D	136. μm²
f/2.8	Canon EOS 1Ds	77.6 μm²
f/4.0	Canon EOS 1DMkII / 5D	67.1 μm²
f/5.6	Nikon D70	61.1 μm²
f/8.0	Canon EOS 10D	
f/11	Canon EOS 1DsMkII	
f/16	Canon EOS 20D / 35	
f/22	Nikon D2X	
f/32	Canon PowerShot G	

Aperture	Camera Type	Pixel Area
f/2.0	Canon EOS 1D	136. μm²
f/2.8	Canon EOS 1Ds	77.6 μm²
f/4.0	Canon EOS 1DMkII / 5D	67.1 μm²
f/5.6	Nikon D70	61.1 μm²
f/8.0	Canon EOS 10D	54.6 μm²
f/11	Canon EOS 1DsMkII	52.0 μm²
f/16	Canon EOS 20D / 350D	41.2 μm²
f/22	Nikon D2X	30.9 μm²
f/32	Canon PowerShot G6	5.46 μm²

(http://www.cambridgeincolour.com/tutorials/diffraction-photography.htm)

The Abbe diffraction limit

$$d = \frac{.61\,\lambda}{NA} \approx 1.2\,N\,\lambda$$

- where
  - λ = wavelength
  - NA = numerical aperture ≈ $1 / 2N$

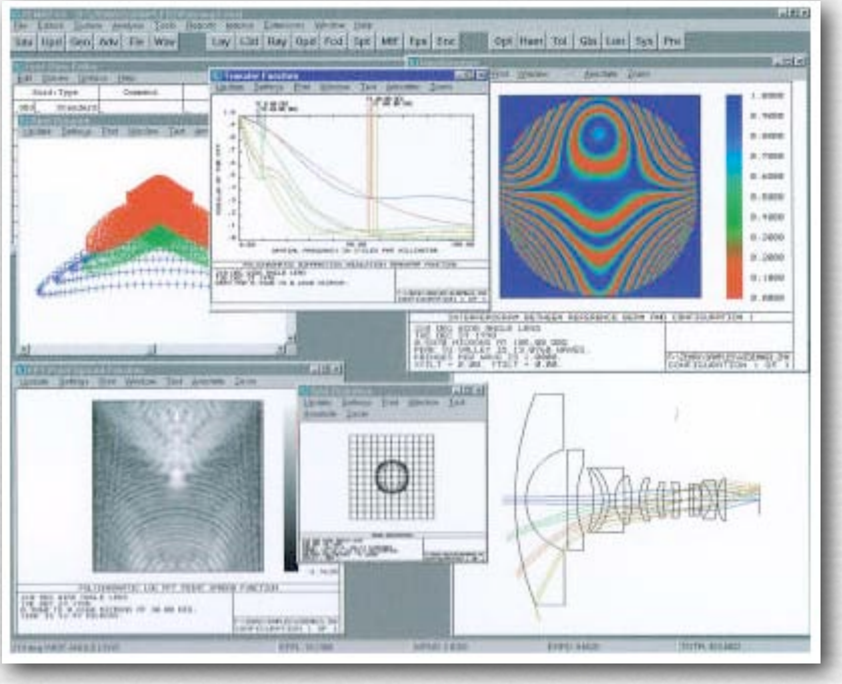
- <u>Example</u>: iPhone 4 when looking at green
  - λ = 550nm
  - N = f/3
  - d = 2μ
  - pixels are 1.75μ wide, so the iPhone 4 would be roughly diffraction-limited if its lenses were free of aberrations

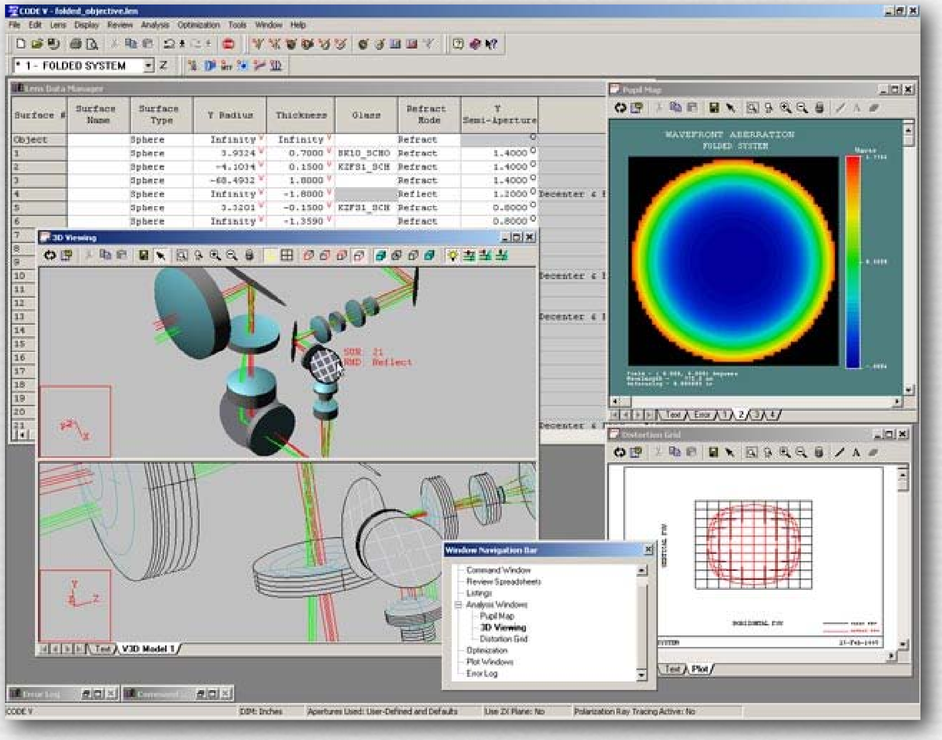
Recap

- all optical systems suffer from veiling glare
  - anti-reflection coatings help
- all optical systems suffer from flare and ghosts
  - don't point your camera at bright lights; use lens hoods
- vignetting arises from many sources
  - natural - falloff at the edges of wide sensors
  - optical - caused by apertures, lens barrels
  - mechanical - caused by wrong lens hoods, hands, straps
  - pixel - caused by shadowing inside pixel structures
- diffraction - blur that varies with $N = f / A$
  - avoid F-numbers above f/13 (for full-frame camera)
  - subjective image quality depends on both sharpness and contrast

Questions?

Lens design software

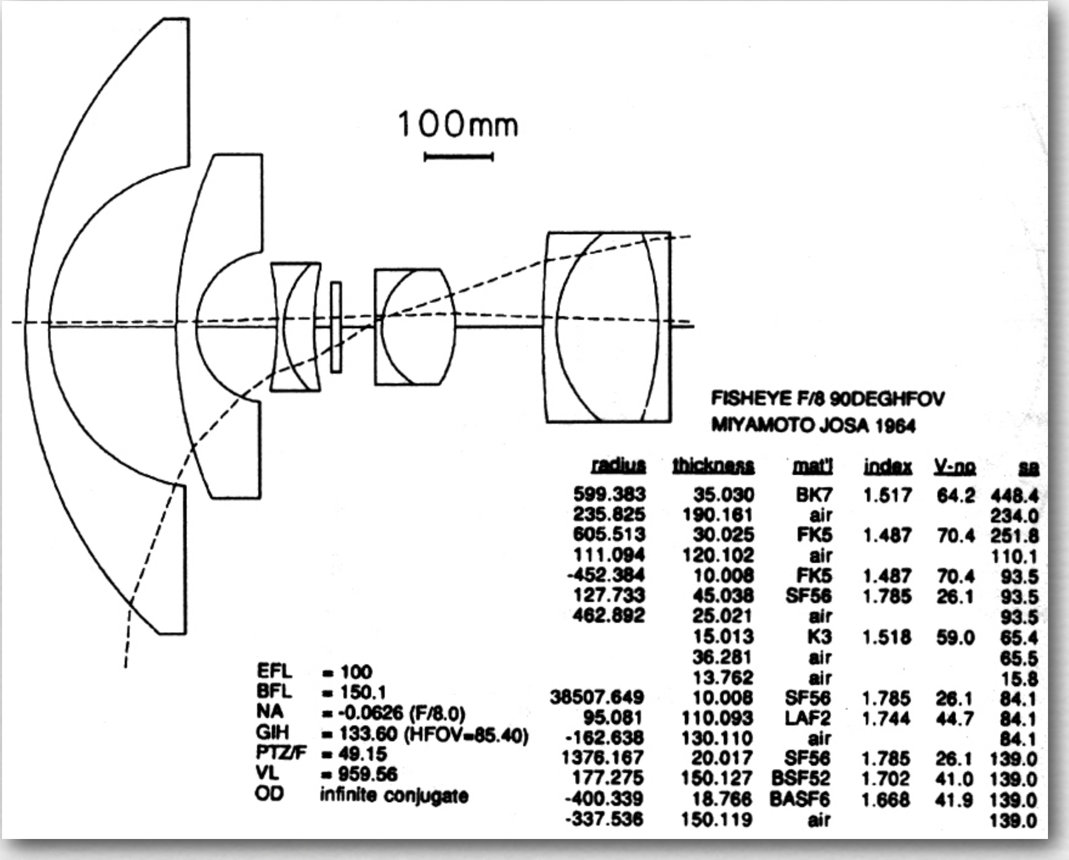




Surface #	Surface Name	Surface Type	Y Radius	Thickness	Glass	Refract Mode	Y Semi-Aperture
Object		Sphere	Infinity	Infinity		Refract	
1		Sphere	3.9324	0.7000	BK10_SCHO	Refract	1.4000
2		Sphere	-4.1014	0.1500	KZFS1_SCH	Refract	1.4000
3		Sphere	-68.4932	1.8000		Refract	1.4000
4		Sphere	Infinity	-1.8000		Reflect	1.2000
5		Sphere	3.3201	-0.1500	KZFS1_SCH	Refract	0.8000
6		Sphere	Infinity	-1.3590		Refract	0.8000

- ✦ uses optimization to make good recipes better

Lens catalogs and patents



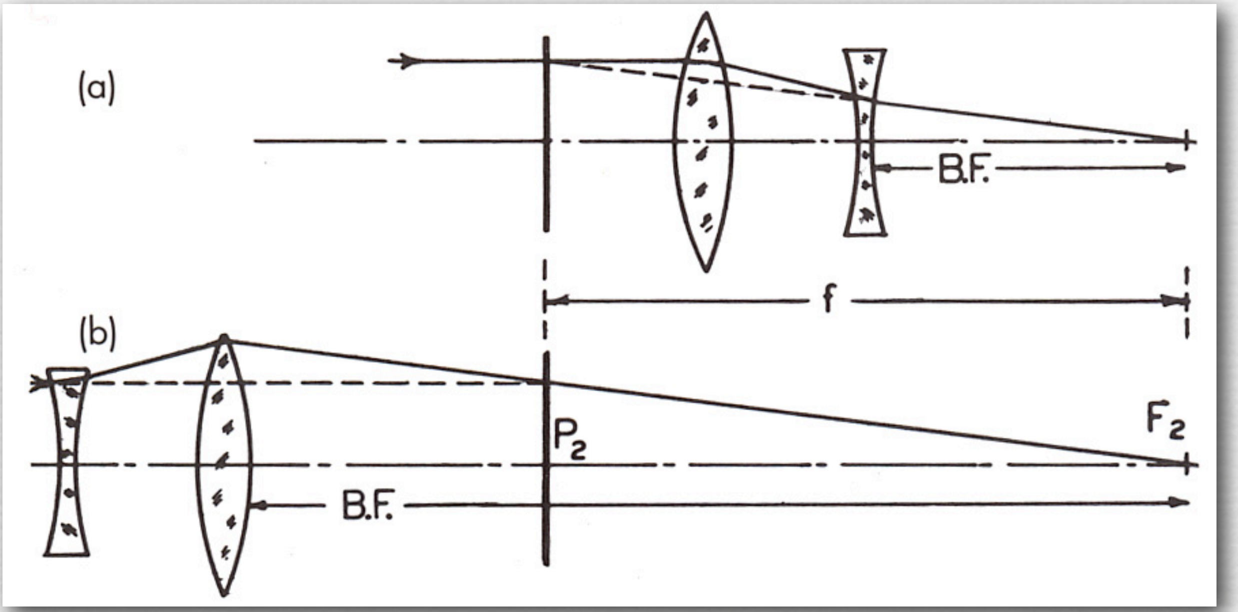
FISHEYE F/8 90DEGHFOV
MIYAMOTO JOSA 1964

radius	thickness	mat'l	index	V-no	sa
599.383	35.030	BK7	1.517	64.2	448.4
235.825	190.161	air			234.0
605.513	30.025	FK5	1.487	70.4	251.8
111.094	120.102	air			110.1
-452.384	10.008	FK5	1.487	70.4	93.5
127.733	45.038	SF56	1.785	26.1	93.5
462.892	25.021	air			93.5
	15.013	K3	1.518	59.0	65.4
	36.281	air			65.5
	13.762	air			15.8
38507.649	10.008	SF56	1.785	26.1	84.1
95.081	110.093	LAF2	1.744	44.7	84.1
-162.638	130.110	air			84.1
1376.167	20.017	SF56	1.785	26.1	139.0
177.275	150.127	BSF52	1.702	41.0	139.0
-400.339	18.766	BASF6	1.668	41.9	139.0
-337.536	150.119	air			139.0

EFL = 100
BFL = 150.1
NA = -0.0626 (F/8.0)
GIH = 133.60 (HFOV=85.40)
PTZ/F = 49.15
VL = 959.56
OD infinite conjugate

- hard to find optical recipe for commercial camera lenses
- patents never contain the real (exact) recipe

Lens combinations: telephoto

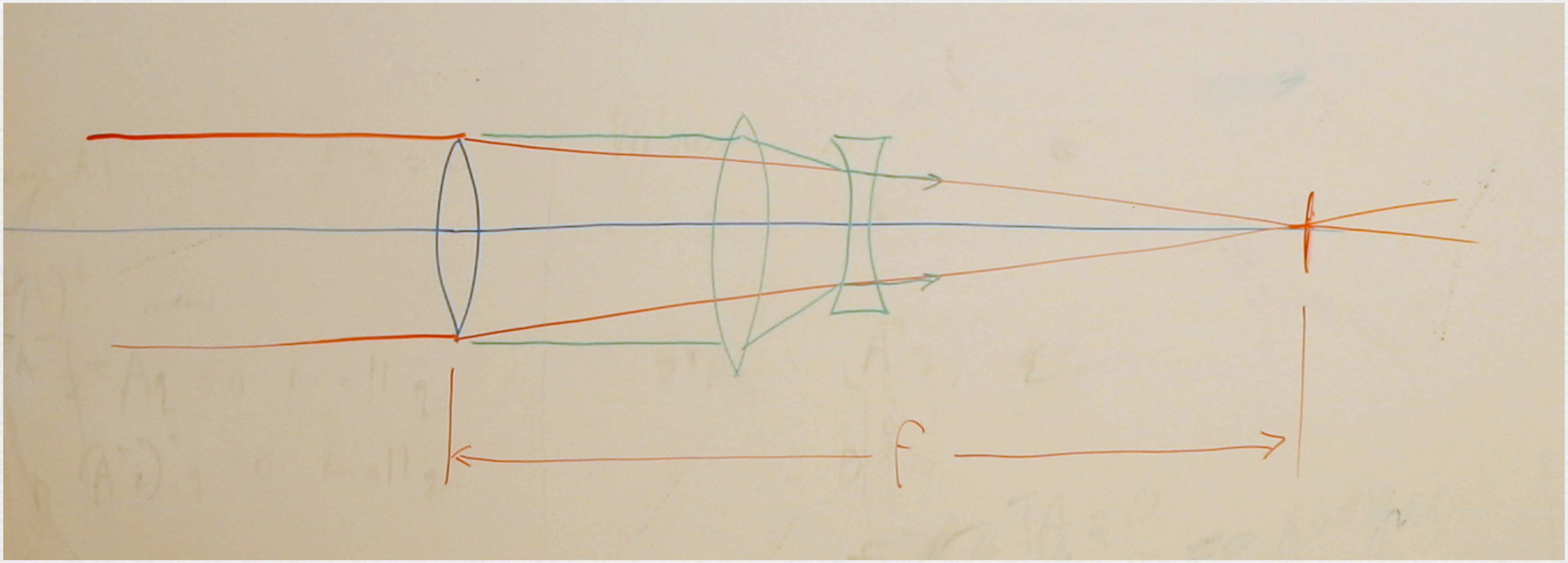


(Kingslake)

- telephoto (a) reduces the back focal distance B.F. relative to *f*
  - for long focal length lenses, to reduce their physical size
- reversed telephoto (b) increases B.F. relative to *f*
  - for wide-angle lenses, to ensure room for the reflex mirror

Telephoto lens

- the blue lens is replaced with the two green ones, thereby reducing the physical size of the lens assembly, while preserving its focal length (hence magnification)

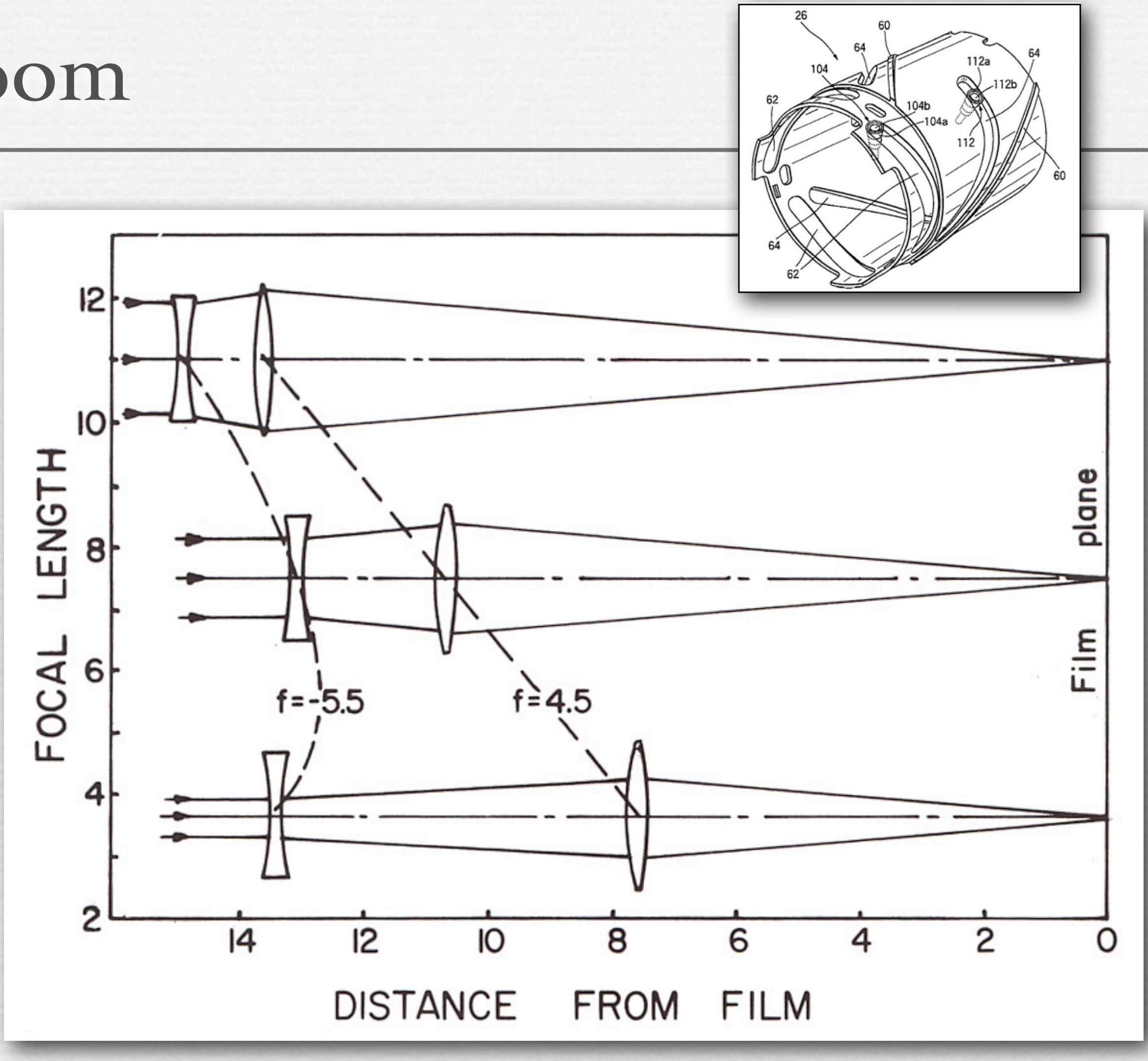


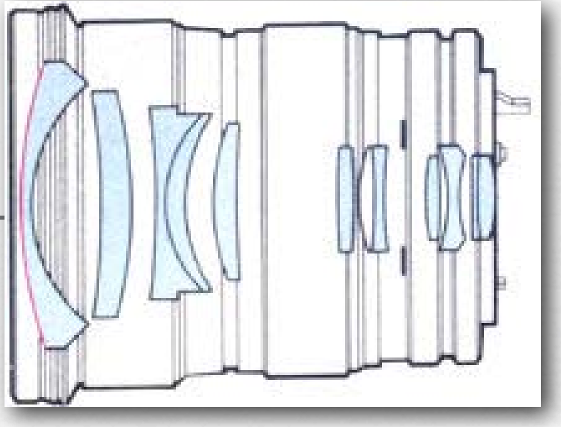
Lens combinations: telephoto

Nikon 500mm telephoto

Opteka 500mm <u>non</u>-telephoto

Lens combinations: zoom





Canon FD 24-35mm
f/3.5 L manual focus lens

(FLASH DEMO)

http://graphics.stanford.edu/courses/cs178/applets/zoom.html

- called *optically compensated zoom*, because the in-focus plane stays (more or less) stationary as you zoom
- to change focus, you move both lenses together

Recap

- telephoto lenses separate focal length & back focal distance
  - for long focal length lenses, to reduce their physical size
  - for wide-angle lenses, to ensure room for the reflex mirror
- most modern zoom lenses are focus-compensated
  - as you zoom, they stay in focus

Questions?

Slide credits

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- Fredo Durand

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